DOCUMENT RESUME

ED 047 979 24 SE 010 842

AUTHOR	Stewart, Deborah Miller
TITLE	Development of a Group Test of Arithmetic Achievement for Developing Mathematical Processes Arithmetic Book 1.
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
REPORT NO	TR-140
BUREAU NO	BR-5-0216
PUB DATE	Aug 70
CONTRACT	OEC-5-10-154
NOTE	147p.; Report from Project on Analysis of Mathematics Instruction, Phase 2
EDRS PRICE	EDRS Price MF-$0.65 HC-$6.58
DESCRIPTORS	Achievement, *Arithmetic, Curriculum Development, *Elementary School Mathematics, *Evaluation, Grade 1, Kindergarten, Mathematical Concepts, Objectives, *Test Construction

ABSTRACT

This master's thesis was designed to construct a group-administered test of kindergarten and first-grade children's knowledge of the content of "Developing Mathematical Processes" Arithmetic Book I. From a list of behavioral objectives for Book I, 15 were selected as ones whose attainment could be measured in a group paper and pencil test. Eight item prototypes were developed from which the items tried out were generated. Items were compiled into seven test booklets each of which was administered to approximately 40 children. Staff members and teachers who had used the instructional program reviewed sample items for their measurement of the objectives they were designed to test. Teachers' comments were generally suggestions for changes in wording; questions raised by staff members could best be answered by research on the completed instrument. Recommendations were made for selection of items to be retained in the completed test. Research to be conducted on the completed instrument was briefly outlined. (Author/FL)

ED047979

BR 5-0216
PA 24
SE

TECHNICAL REPORT

U.S. DEPARTMENT OF HEALTH, EDUCATION
& WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

THE WISCONSIN RESEARCH
AND DEVELOPMENT CENTER
FOR COGNITIVE LEARNING

PE 010 842

ED047979

Technical Report No. 140

DEVELOPMENT OF A GROUP TEST OF ARITHMETIC ACHIEVEMENT FOR DEVELOPING MATHEMATICAL PROCESSES ARITHMETIC BOOK I

Report from the Project on Analysis of
Mathematics Instruction,
Phase 2

by Deborah Miller Stewart

T. Anne Cleary, Associate Professor of Educational Psychology
Chairman of the Examining Committee

Thomas A. Romberg and John G. Harvey
Principal Investigators

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin

August 1970

This Technical Report is a master's thesis reporting research supported by the Wisconsin Research and Development Center for Cognitive Learning. Since it has been approved by a University Examining Committee, it has not been reviewed by the Center. It is published by the Center as a record of some of the Center's activities and as a service to the student. The bound original is in The University of Wisconsin Memorial Library.

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

NATIONAL EVALUATION COMMITTEE

Samuel Brownell
Professor of Urban Education
Graduate School
Yale University

Launor F. Carter
Senior Vice President on
Technology and Development
System Development Corporation

Francis S. Chase
Professor
Department of Education
University of Chicago

Henry Chauncey
President
Educational Testing Service

Martin Deutsch
Director, Institute for
Developmental Studies
New York Medical College

Jack Edling
Director, Teaching Research
Division
Oregon State System of Higher
Education

Elizabeth Koontz
Wage and Labor Standards
Administration, U.S.
Department of Labor,
Washington

Roderick McPhee
President
Punahou School, Honolulu

G. Wesley Sowards
Director, Elementary Education
Florida State University

Patrick Suppes
Professor
Department of Mathematics
Stanford University

***Benton J. Underwood**
Professor
Department of Psychology
Northwestern University

RESEARCH AND DEVELOPMENT CENTER POLICY REVIEW BOARD

Leonard Berkowitz
Chairman
Department of Psychology

Archie A. Buchmiller
Deputy State Superintendent
Department of Public Instruction

Robert E. Grinder
Chairman
Department of Educational
Psychology

Russell J. Hosler
Professor, Curriculum
and Instruction

Clauston Jenkins
Assistant Director
Coordinating Committee for
Higher Education

Herbert J. Klausmeier
Director, R & D Center
Professor of Educational
Psychology

Stephen C. Kleene
Dean, College of
Letters and Science

Donald J. McCarty
Dean
School of Education

Ira Sharkansky
Associate Professor of Political
Science

B. Robert Tabachnick
Chairman, Department
of Curriculum and
Instruction

Henry C. Weinlick
Executive Secretary
Wisconsin Education Association

M. Crawford Young
Associate Dean
The Graduate School

EXECUTIVE COMMITTEE

Edgar F. Borgatta
Brittingham Professor of
Sociology

Anne E. Buchanan
Project Specialist
R & D Center

Robin S. Chapman
Research Associate
R & D Center

Robert E. Davidson
Assistant Professor,
Educational Psychology

Frank H. Farley
Associate Professor,
Educational Psychology

Russell J. Hosler
Professor of Curriculum and
Instruction and of Business

***Herbert J. Klausmeier**
Director, R & D Center
Professor of Educational
Psychology

Wayne Otto
Professor of Curriculum and
Instruction (Reading)

Robert G. Petzold
Associate Dean of the School
of Education
Professor of Curriculum and
Instruction and of Music

FACULTY OF PRINCIPAL INVESTIGATORS

Vernon L. Allen
Professor of Psychology

Ted Czajkowski
Assistant Professor of Curriculum
and Instruction

Robert E. Davidson
Assistant Professor of
Educational Psychology

Gary A. Davis
Associate Professor of
Educational Psychology

M. Vere DeVault
Professor of Curriculum and
Instruction (Mathematics)

Frank H. Farley
Associate Professor of Educational
Psychology

Lester S. Golub
Lecturer in Curriculum and
Instruction and in English

John G. Harvey
Associate Professor of
Mathematics and of Curriculum
and Instruction

Herbert J. Klausmeier
Director, R & D Center
Professor of Educational
Psychology

Donald Lange
Assistant Professor of Curriculum
and Instruction

James Moser
Assistant Professor of Mathematics
Education; Visiting Scholar

Wayne Otto
Professor of Curriculum and
Instruction (Reading)

Milton O. Pella
Professor of Curriculum and
Instruction (Science)

Thomas A. Romberg
Associate Director, R & D Center
Professor of Mathematics and of
Curriculum and Instruction

B. Robert Tabachnick
Chairman, Department
of Curriculum and
Instruction

Richard L. Venezky
Assistant Professor of English
and of Computer Sciences

Alan Voelker
Assistant Professor of Curriculum
and Instruction

Larry Wilder
Assistant Professor of Curriculum
and Instruction

Peter Wolff
Assistant Professor of Educational
Psychology

MANAGEMENT COUNCIL

Herbert J. Klausmeier
Director, R & D Center
V.A.C. Henmon Professor of
Educational Psychology

Mary R. Quilling
Director
Technical Development Program

Thomas A. Romberg
Associate Director

James Walter
Director
Dissemination Program

Dan G. Woolpert
Director
Operations and Business

* COMMITTEE CHAIRMAN

STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related education practices. The strategy for research and development is comprehensive. It includes basic research to learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathematics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

CONTENTS

Page

Abstract ix

I. Background and Statement of the Problem 1

II. Test Development 12

Preparation of Content Outline 12

Item and Test Specifications 14

Item Tryouts 22

Scoring 25

Content Validity 27

Data Analysis 27

III. Results 28

Feasibility of the Instrument 28

Content Validity 29

Item Analysis 33

Tests I and II, Classifying and Comparing Objects and Objects and Sets 36

Test III, Equalizing Objects on Length . . . 41

Test IV, Equalizing Sets on Numerousness . . 43

Test V, Ordering More Than Two Objects or Sets 46

Tests VI and VII, Numbers and Numerals . . . 49

IV. Recommendations 53

Recommendations for Specific Tests 53

Tests I and II, Classifying and Comparing Objects and Sets 53

Tests III and IV, Equalizing Objects and Sets 53

CONTENTS Continued

Page

Test V, Ordering 55

Recommendations for the Entire Battery 55

Mechanical 55

Technical 57

References 60

Appendix A: Test Directions 62

Appendix B: Tests Administered in Item Tryouts . . 86

Appendix C: Materials Distributed for Content Validity Study 184

Appendix D: Item Statistics 220

LIST OF TABLES AND FIGURES

Table Page

1 Topic Outline and Objectives of Developing Mathematical Processes Arithmetic Book I 7

2 Test Development Guide 15

3 Item Prototypes 16

4 Revised Test Development Guide 20

5 Number of Subjects Taking Each Test 23

6 Summary of Mean Scores, Reliabilities, and Related Statistics for All Seven Tests 34

7 Estimated Reliabilities for Tests of Different Lengths 37

8 Summary of Statistics for Tests I and II, Classifying and Comparing Objects and Sets, Total Test and Subscales 38

9 Summary of Statistics for Test III, Equalizing Objects on Length, Total Test and Subscales 42

10 Summary of Statistics for Test IV, Equalizing Sets on Numerousness, Total Test and Subscales 44

11 Summary of Statistics for Test V, Ordering More Than Two Objects or Sets, Total Test and Subscales 49

12 Summary of Statistics for Tests VI and VII, Numbers and Numerals, Total Test and Subscales 50

Figure

1 Major Components of the Task Analysis for Developing Mathematical Processes Arithmetic Book I 6

2 (a) Correct and (b) Half-Right Answers to Test III 26

ABSTRACT

Constructing a group-administered test of kindergarten and first-grade children's knowledge of the content of Developing Mathematical Processes Arithmetic Book I was the purpose of the study. From the list of behavioral objectives for Book I, 15 were selected as ones whose attainment could be measured in a group paper and pencil test. Eight item prototypes were developed from which the 171 items tried out were generated. Items were compiled into seven tests (booklets), each of which was administered to approximately 40 children. Staff members and teachers who had used the instructional program reviewed sample items for their measurement of the objectives they were designed to test. Teachers' comments were generally suggestions for changes in wording, most of which were implemented in item tryouts; questions raised by staff members could best be answered by research on the completed instrument. Recommendations were made for selection of items to be retained in the completed test; research to be conducted on the completed instrument was briefly outlined.

I

BACKGROUND AND STATEMENT OF THE PROBLEM

The contrast between young children unafraid of failure in a search for new information and students striving desperately for a way to win the teacher's game of right answers is one of the many contrasts between tomorrow's elementary school and the typical school of today. Today's schools, those that "have not incorporated such practices as team teaching, preservice teacher education, inservice education, or systematic instructional improvement," are trying only to maintain "an instructional program that is as good as in the past (Klausmeier, Morrow, Walter, & Way, 1968, p. 9)." In the emerging schools, knowledge of the characteristics of each student and of the educational objectives and program of the school will be combined to provide individually guided education, instructional programs that meet individual needs.

Evaluation will have a new role in individually guided education, a role that brings it out of the cumulative record and into daily planning. As a process that is "inherent in every teaching activity (Klausmeier & Dresden, 1962, p. 157)," evaluation is carried out in both schools of today and those of tomorrow. Evaluation of students may be defined, adapting Stake's definition of curriculum evaluation (1967, p. 5), as the "collection, processing, and interpretation of data pertaining to" a child. On a school-wide or system-wide basis the data are results of standardized and teacher-developed tests that have typically been used in making comparisons among students, in assigning grades, and in making comparisons among schools. Where the system of individually guided education is adopted, results of the same tests and other procedures will be used to determine each child's initial characteristics and his progress, to notify each child and his teachers of the progress he is making, and to improve the instructional program (Klausmeier et al., 1968).

While observing students at work, talking with them, scoring their papers and tests, or performing others of the many instructional tasks, each teacher builds a repertoire of knowledge about students' abilities and their use of them, thus carrying out the collection and processing of data in the instructional group. Interpretation of the data is completed as the teacher applies the accumulated knowledge to instructional planning either for an entire class or for the individual student.

Planning for the entire class, customary in today's schools, has led to the result noted by Tyler (1967) that of all students there are "15 to 20 per cent who have not been making appreciable progress in learning (p. 15.)" He stated that the percentage could be reduced although it would be necessary to have new educational concepts which are "based on the assumption of dynamic potential in all or almost all human beings. The evaluation task is to describe or measure phases of this potential and difficulties to be surmounted that can help the individual and the educational institution in improving learning (p. 16)." Bloom (1968) also felt that schools' expectations were too low; he reported that typically only two-thirds of the students learn even a "good deal" of what is taught, while it is possible that "given sufficient time (and appropriate types of help), 95 percent of students . . . can learn a subject up to a high level of mastery." The procedure Bloom followed in attempting to reach such a high percentage of students included analyzing content into a hierarchy of learning tasks, providing brief diagnostic-progress tests, and prescribing study based on the results of the test administration. At the Wisconsin Research and Development Center for Cognitive Learning curriculums appropriate for individually guided education are being prepared. One of these programs is Developing Mathematical Processes (DMP), a mathematics instruction program for the elementary school, whose elements are developed through a process similar to that followed by Bloom.

Subsequent to definition of content, a task analysis is performed, using the techniques first described by Gagne (Gagne & Paradise, 1961; Gagne, Mayor, Garstens, & Paradise, 1962), to identify prerequisite behaviors for each concept and to analyze each concept "in terms of its subconcepts, properties, or attributes, together with any rules for combining them (Romberg, Fletcher, & Scott, 1968, p. 6)." The completed task analysis provides a logical hierarchy of concepts for a unit of instruction; a subsequent instructional analysis results in a linear sequence of topics based on the hierarchy. Progress tests designed for administration after completion of specified segments of instruction have most often been individually administered performance tests. Teachers were encouraged to use the results of these tests in planning instructional activities, providing remedial work for students not performing at a satisfactory level, and guiding exploratory work for those students whose performance was satisfactory or better than required.

A mathematics educator, Thomas A. Romberg, and a developmental psychologist, Harold J. Fletcher, initiated Developing Mathematical Processes. They were directing the Analysis of Mathematics Instruction Project whose goals were to focus on analysis and research of mathematics instruction and to find the fundamental psychological units, concepts, and skills necessary for learning mathematics; the investigators felt that DMP would provide an effective means of synthesizing results of the Project's activities and making them available to the educational community. Professor Fletcher is now a consultant to the Project; a mathematician, John G. Harvey, replaced him as one of the Project's directors.

Six subject areas have been identified for inclusion in the completed instructional program: the arithmetic of rational numbers; the geometry of physical time-space; the fundamentals of probability and statistics; the use of logic, reasoning, and proof; problem-solving strategies; and the encouragement of mathematical creativity. Instructional units making up a K-6 program have been defined for the areas of arithmetic and geometry and materials have been prepared for arithmetic instructing during approximately the first three years, K-2. Arithmetic Book I, Comparing and Equalizing Objects and Sets, is used as the initial arithmetic text in the elementary school, typically during kindergarten. In Figure 1, a simplification of the hierarchy produced from the task and instructional analyses for Book I is presented; from this hierarchy, the topics and behavioral objectives listed in Table 1 were developed. As indicated in the table, individual tests have been designed for administration at the completion of Topics 2, 7, 11, 14, and 16 of Book I.

Book I was designed so that the activities outlined for instruction would provide children with the methods for independent verification of their work when they perform mathematical operations with numerals. The initial topics of the book deal with comparing and classifying objects on any properties identified by the students, typically extrinsic properties, as well as intrinsic properties introduced by the teacher if necessary. Children's interest in themselves is used as the basis for many classifying activities. Brightly colored objects that differ in length are used in the first length comparisons; children later use these same objects as arbitrary units in measuring lengths that cannot be directly compared. Sets, too, are compared on many properties before the property of numerousness is defined. A baseline and one-to-one matching are introduced as necessities for proper comparisons of objects and sets respectively. Children make picture graphs, bar graphs, and tally marks to record the results of their activities, thus using representations less abstract than numerals as they gain experience with the equalizing process so basic to elementary arithmetic.

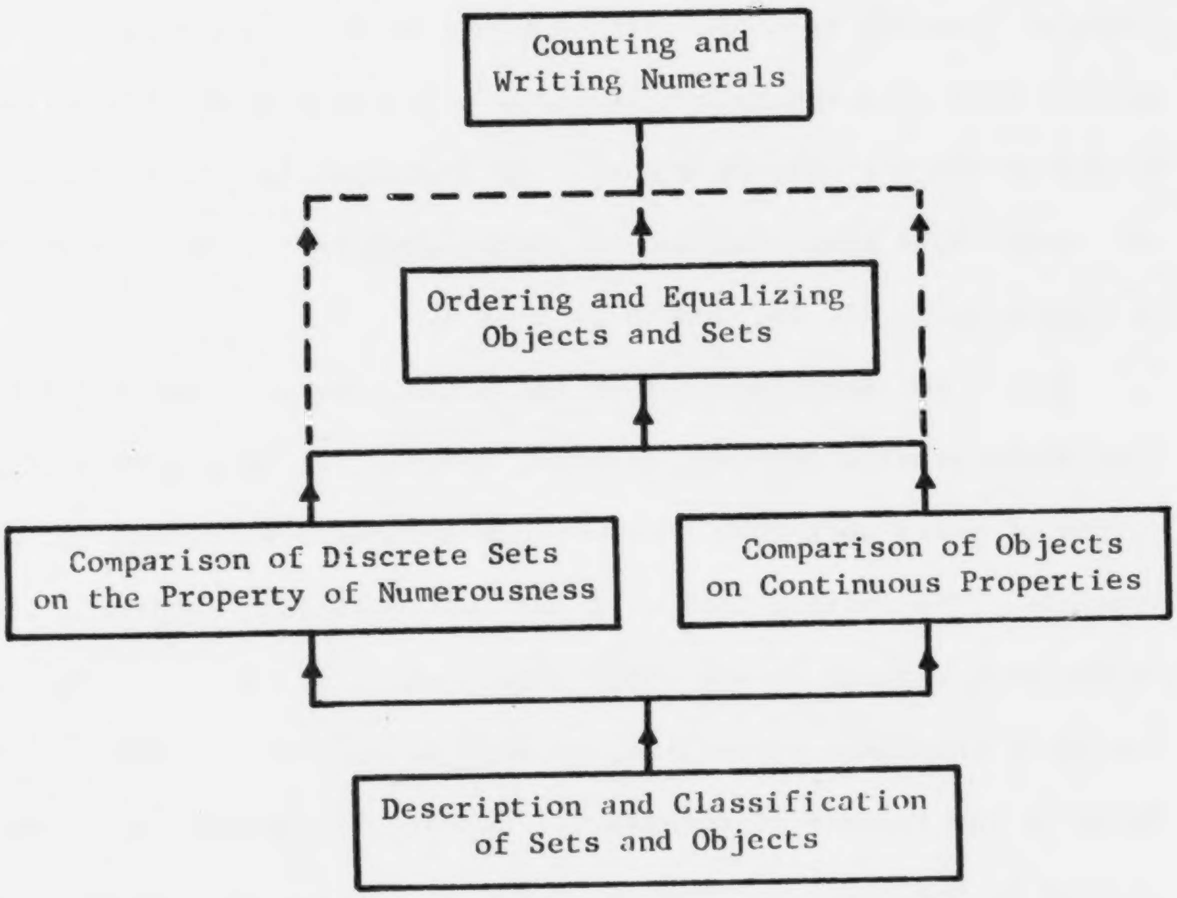

Counting and Writing Numerals
Ordering and Equalizing Objects and Sets
Comparison of Discrete Sets on the Property of Numerousness
Comparison of Objects on Continuous Properties
Description and Classification of Sets and Objects

Fig. 1.--Major components of the task analysis for Developing Mathematical Processes Arithmetic Book I.

(Adapted from Romberg, Harvey, and McLeod, 1970)

Table 1

Topic Outline and Objectives of *Developing Mathematical Processes* Arithmetic Book I

Objective No.	Objective
Topic 1,	Identifying Properties of Objects
1	Identify objects
2	Differentiate among properties of objects
3	State the property values of objects
4	State a specific property value which several objects have in common
Topic 2,	Classifying and Describing Objects
1	State how objects are alike or different when focusing on a single property
2	(a) Separate objects into two sets, positive and negative instances of a single property value, and (b) state the classification rule used
3NM*	Separate objects into sets as determined by the presence of two or more specific property values and state the classification rule used
4NM	Discriminate among, identify, and classify geometric properties of objects
5	State the classification rule used when shown sets of objects
	(First Individual Test)
Topic 3,	Identifying Length as a Property of Objects and Comparing Objects on Length
1	Identify length as a property on which objects can be compared
2	Place two objects side by side to determine whether they are the same length
3	State the length relationship between two objects of unequal length
Topic 4,	Equalizing Objects on Length
	Equalize the length of objects which are unequal, either by "taking away" from the longer object or by "adding to" the shorter object
Topic 5,	Ordering Objects on Length
1	Order more than two objects from shortest to longest
2	Order more than two objects from longest to shortest

(Continued)

Table 1 (cont'd.)

Objective No.	Objective
Topic 6,	Representing Length Physically
1	Make accurate physical representations of the length of at least one of two objects
2	Compare, equalize, and order representations of length
Topic 7,	Representing Length Pictorially
1	Make pictorial representations of length
2	Compare, equalize, and order representations of length
	(Second Individual Test)
Topic 8,	Classifying and Describing Sets of Objects
1	State how sets are alike or different when focusing on a single property
2	(a) Separate sets into two groups, positive and negative instances of a single property value, and (b) state the classification rule used
3NM	Separate sets into two groups as determined by the presence of two or more property values and state the classification rule used
Topic 9,	Identifying Numerousness as a Property of Sets and Comparing Sets on Numerousness
1	Verbally identify numerousness as a property on which two sets can be compared
2	Compare the numerousness of two sets using one-to-one matching to determine whether they are the same size
3	State the numerousness relationship which exists between two sets of unequal size
Topic 10,	Equalizing Sets on Numerousness
	Equalize any two sets of objects which are unequal in number by either adding more objects to the smaller set or by taking objects away from the larger set
Topic 11,	Ordering Sets on Numerousness
1	Order more than two sets on numerousness from largest to smallest and state order relationship used
2	Order more than two sets on numerousness from smallest to largest and state order relationship used
	(Third Individual Test)
Topic 12,	Representing Numerousness Physically
1	Make physical representations of the numerousness of sets
2	Compare, equalize, and order representations of the numerousness of sets

(Continued)

Table 1 (cont'd.)

Objective No.	Objective
Topic 13,	Representing Numerousness Pictorially
1	Make pictorial representations of the numerousness of sets
2	Compare, equalize, and order representations of sets
Topic 14,	Tallying Units of Length
1NM	Choose an arbitrary unit of length
2	Measure a specified object using the arbitrary unit of length
3	Make tally marks to represent the number of arbitrary units required to equal the length of the object
	(Fourth Individual Test)
Topic 15,	Counting from 0 to 10
1	Count up to ten objects in a set
2	Represent the numerousness of a set composed of 0-10 objects by stating the appropriate number
3	Illustrate or display a set of objects the number of which is specified by a stated number from zero to ten
Topic 16,	Recognizing the Numerals 0-10
1	State the number represented by a written symbol
2	Select and place in the correct position the appropriate numeral to represent a given number
3	Illustrate or select a set of objects the number of which is represented by a given numeral
4	Select the appropriate numeral to represent a given set of objects
5	Compare and order the numerals from 0 to 10
	(Fifth Individual Test)

*Indicates nonmastery objective

As the available elements of the curriculum have been used in the schools, it has become apparent that individual evaluations, although quite useful for identifying specific weaknesses and designing particular instructional sequences, do not meet all of the teacher's needs. In particular, teachers need to know fairly soon after the beginning of the school year the level at which various students are able to operate within the mathematics curriculum to group children for instruction. If small groups of their students were working at the same pace, teachers might want to determine which, if any, segments of the curriculum should be reviewed before the next unit is begun.

The test of greatest aid to teachers making instructional decisions like those just mentioned would be a reliable one whose content had been specified to represent the curriculum used; efficiency, i.e., group administration over a brief period of time, should also be a characteristic of the instrument. To construct such a test for teachers using Arithmetic Book I, the first element of DMP, is the purpose of this study. The completed test will be part of an evaluation package designed to provide guidelines for data interpretation as well as the means for their collection and processing in a curriculum designed for individually guided education.

Because the content of Book I is considered hierarchical in nature, from the instrument to test mastery of that content a profile of scores, rather than a total score, should be obtained for each student. Anastasi (1968) differentiated between survey tests which "indicate the general level of the individual's achievement" and diagnostic tests which are "designed to analyze the individual's strengths and weaknesses in the area and to suggest causes of his difficulty (p. 404)" and stated that several scores were obtained from a diagnostic test. Cronbach (1960), too, emphasized the pattern of scores that result from administration of a diagnostic test; he seemed to be writing of tests related to a hierarchically ordered curriculum when he stated that "the tester is left with a picture of the specific weaknesses that must be remedied before the pupil can make normal progress (p. 390)." Although the planned brevity of the group test being developed will probably prohibit identification of specific weaknesses or causes of difficulty, study of a profile of the scores would aid in identifying those students whose learning difficulties need to be more closely examined.

II

TEST DEVELOPMENT

PREPARATION OF CONTENT OUTLINE

From the complete set of behavioral objectives for Arithmetic Book I (Table 1),, the following thirteen, listed by topic number and objective number within topic, were selected as ones whose attainment could be measured in a group paper-and-pencil test: 2.2a, 3.3, 4.-, 5.1, 5.2, 8.2a, 9.3, 10.-, 11.1, 11.2, 15.3, 16.3, and 16.4. In the paragraphs below, the basis for selection of these objectives is given.

Because they require that children make an oral response to demonstrate mastery, the following objectives were omitted from consideration as measurable in the test to be developed: 1.1, 1.3, 1.4, 2.1, 2.2b, 2.5, 8.1, 8.2b, 3.1, 9.1, 15.1, 15.2, and 16.1. Objectives 3.3 and 9.3 appear to require statements but can be rephrased to require only a differentiation between two alternatives; for example, "State the length relationship between two objects of unequal length," Objective 3.3, could also be read "Select the longer (shorter) of two objects of unequal length."

Ability to construct and work with physical and pictorial representations of objects and sets was considered not measurable in this unit test; objectives of Topics 6, 7, 12, 13, and 14, thus, were removed from consideration. The performance of first-grade children in individual testing earlier in the year contributed to the omission of representations: Some children constructed bar graphs with primary attention to coloring precision and could, in a group test, either not complete the task or hold up the entire group; when tallying was the desired response, some children counted, giving an answer that, although more abstract than required, did not indicate whether they could use tally marks as representations. Most important is the fact that many children who have not received the instruction based on Arithmetic Book I will have independently developed a sufficient understanding of numbers and number concepts to respond correctly to many items; the construction of graphs and other representations is a skill typically taught in the intermediate grades (e.g., DeVault, Osborn, & Forester, 1966; Eicholz & O'Daffer, 1968; Hartung, Van Engen, Gibb, Stochl, Knowles, & Walch, 1967) and not necessarily in the repertoire of a child who has the mathematical understanding required for further instruction based on DMP.

The preceding two paragraphs describe the primary steps in omitting objectives; other considerations led to the deletion of a few objectives each. Nonmastery objectives, indicated by NM in Table 1, were not considered in test development. Objective 1.2 was considered to be such a basic requirement that a child not meeting that objective would be unable to respond to nearly all of the test items; it was not to be directly measured. Correct position or orientation of the numerals, Objective 16.2, was considered of primary importance for the later writing of numerals and most meaningfully measured immediately prior to writing by an individual observation which used numerals that could be placed backwards, sideways, etc. The instructional emphasis given to Objective 16.5 in Arithmetic Book II was considered sufficient reason for omitting that objective.

Objectives 3.2 and 9.2 relating to the arrangement of objects and sets for comparison were omitted after prototype items were constructed. The prototype items for these objectives appeared to require such an elementary response that most children would learn the techniques from sample items well enough to make satisfactory responses but might not exhibit the proper behavior when given actual objects or sets. After item specifications had been written, it was noted that there were not items to measure children's ability to discriminate between equal and unequal objects (lengths) or sets. The omission was traced to the wording of Objectives 3.2 and 9.2; items picturing two objects of equal length and items picturing two sets equal in number were developed.

ITEM AND TEST SPECIFICATIONS

Seven item types were defined to measure children's attainment of the thirteen selected objectives. Percentages of the total number of items were allotted to the item types; these weights are listed in Table 2 along with the item types and the objectives they relate to. Prototype items and directions are given in Table 3 in lieu of descriptions of the item types. For ease of reading, Prototypes 3-5 are written in terms of objects only, rather than objects and sets; by substitu-

Table 2

Test Development Guide

Objective number	Item type	Weighting[a]		No. items tried out
		Type	Obj.	
	1. Classifying	15		
2.2a	Objects		7	12
8.2a	Sets		8	12
	2. "Stating" relationship between two	6		
3.3 (3.2b)	Objects		3	8(12)[b]
9.3 (9.2b)	Sets		3	8(12)[b]
	3. Equalizing	29		
4.-	Objects		14	22
10.-	Sets		15	22
	4. Ordering	15		
5.1, 5.2	Objects		8	12
11.1, 11.2	Sets		7	11
15.3	5. Selecting set for stated number	10	10	16
16.3	6. Selecting set for numeral	10	10	16
16.4	7. Selecting numeral to represent set	15	15	24
	Total	100	100	163(171)[b]

[a] Read either as percent or, since a 100-item test is sought, as number of items.

[b] Parentheses indicate change from number planned to be tried out because of initial omission of discrimination between equal and unequal objects and sets.

ting sets for objects and the property of numerousness for length, the reader can realize the prototypes' applicability to both objects and sets.

Table 3

Item Prototypes

Type No.	Child sees	Teacher reads
1	Picture of two or more objects or sets	Put an X on the objects [sets] that have specific property value.
2	Picture of two objects side by side or of two sets	If the objects are equal in length [sets are equal in number], put an X on both objects [sets]. If the objects are not equal in length [sets are not equal in number], put an X on the longer (shorter) object [the set that is larger (smaller) in number].
3	Picture of two objects that are unequal in length placed side by side	Use your crayon to show how much you would add to the shorter (take away from the longer) name of object to make the name of objects equal in length.
4	Three to five objects unequal in length and a square for each object. One of the end squares may be marked with an X or a line may be drawn from the longest or shortest object to one of the end squares and the next square marked with an X .	Draw a line from the box with an X in it to the longest (shortest) name of object. OR One of the name of objects has been put in order by length. Put another name of object in order by length. Draw a line from the box with an X in it to the name of object that belongs in that box.
5	Picture of ten objects	Draw a circle around a set of any number from 0 to 10 objects.
6	Numeral from 0 to 10 and picture of ten objects	Look at the numeral on this page. Draw a circle around a set of as many name of objects as the numeral says.
7	All numerals from 0 to 10 and from 0 to 10 objects	Draw a circle around the numeral that tells how many name of objects there are on this page.

Direction and a set of sample items, one for each objective, were prepared and distributed to members of the Project staff for review. After revisions in the items and directions had been made, a former first-grade teacher, a technical specialist familiar with testing, was invited to review the items. A final review of sample items and directions was made by Margaret L. Harris, measurement consultant at the R & D Center. When all queries from reviewers had been resolved, specifications for all test items were written and submitted for review by the Project's evaluation specialist. Seven test booklets were planned with the following descriptive titles:

I Classifying and Comparing Objects and Sets I
II Classifying and Comparing Objects and Sets II
III Equalizing Objects on Length
IV Equalizing Sets on Numerousness
V Ordering More Than Two Objects or Sets
VI Counting and Recognizing Numerals I
VII Counting and Recognizing Numerals II

Booklets I and II and Booklets VI and VII were planned to be parallel to maintain the maximum number of items per booklet at approximately 25 and to secure data on the necessary number of items.

Names of the objects pictured were checked for frequency of occurrence in two work lists. Occurence either more than 100 times per million printed words, AA rating in the Thorndike and Lorge (1944) word list, or in the first ttwo thousand most frequently used words in first graders' writing samples (Rinsland, 1945) or in both lists was considered appropriate. Of the objects classified by the children and not named by the test administrator, over 90% had names meeting the criteria of acceptance. Names of other objects--cupcake, pillow, mitten, screw, skillet, teakettle--were considered to be familiar enough that the pictures could appear in the items tried out. Since many objects are identified by the test administrator and appear in the items as, for example, sets to be assigned a number or objects to be made equal in length, the overall rate of 88% of the names meeting one or both criteria was considered satisfactory.

In Booklets III and IV, the order of instructions to add or take away to equalize the objects or sets was determined from the order of odd and even numbers in a table of random numbers with the restrictions that no more than two add or take away items occurred in succession and that there be eleven items for each method of equalizing. A difference of no more than four objects between the sets in Booklet IV was considered sufficient to determine whether children could equalize sets and small enough that responses would not require a great deal of time. From a table of random numbers, the numerousness of each pair of sets was determined, maintaining the +/- restriction previously developed; empty sets were arbitrarily excluded as were sets of ten members. In the 56 items comprising Booklets VI and VII, each of the numbers or numerals from 0 to 10 was to appear five times, the numeral appearing six times being determined by a table of random numbers. Within each of the three types of items, all eleven numbers were to be represented at least once and no number would be represented three times unless all could appear twice.

After test development had been completed, Item Type 4 was felt to be of much greater difficulty mechanically than the other five types. Two less complex prototypes were developed after the pairs of Objectives 5.1 and 5.2 and Objectives 11.1 and 11.2 were restated to read as follows: (1) Identify the longest and shortest or largest and smallest in number from more than two objects or sets and (2) order more than two objects or sets. The two prototype items replacing the single type originally devised are given below.

4a. Child sees three or four objects that are unequal in length placed side by side.
Teacher reads: Put an X on the longest (shortest) name of object.

4b. Child sees three to five objects unequal in length placed side by side. Space is shown for all to be placed in order by length and one or two objects are in place; others are at side or bottom of page depending on whether the comparison is of horizontal or vertical length.
Teacher reads: One (Some) of these name of objects has (have) been put in order by length. Put another object in order by length. Put an X on the name of object at the bottom (side) of the page that goes next in order by length.

Items based on Type 4a could be administered with those of Types 1 and 2 as the same response is asked of the children; at the time revisions were made, booklets in which Types 1 and 2 appear had been printed so items like Type 4a were administered with those like Type 4b. Of the 23 items originally developed from Type 4, four relating to objects and four to sets were developed from Type 4a with the remaining items developed from Type 4b. After these changes in the item prototypes were made, the test development guide was revised as shown in Table 4.

Two limits were imposed to define the maximum test length of 100 items. First, five- and six-year-olds should not be expected to work more than one half-hour at a time; former teachers who reviewed the sample items estimated that children could respond to approximately

Table 4

Revised Test Development Guide

Objective number	Item type	Weighting[a] Type	Weighting[a] Obj.	No. items tried out
	1. Classifying	15		
2.2a	Objects		8	12
8.2a	Sets		7	12
	2. "Stating" relationship between two	6		
3.2b, 3.3	Objects		3	12
9.2b, 9.3	Sets		3	12
	3. Equalizing	28		
4.-	Objects		14	22
10.-	Sets		14	22
	4a. Identifying from more than two	4		
5.1[b]	Objects, longest and shortest		2	4
11.1[b]	Sets, largest and smallest in number		2	4
	4b. Ordering more than two	12		
5.2[b]	Objects		6	8
11.2[b]	Sets		6	7
15.3	5. Selecting set for stated number	10	10	16
16.3	6. Selecting set for numeral	10	10	16
16.4	7. Selecting numeral to represent set	15	15	24
	TOTAL	100	100	171

[a] Read either as percent or, since a 100-item test is sought, as number of items.

[b] Objective restated.

20-25 items in 30 minutes. Second, teachers are not likely to be willing to administer tests for more than one school week before instruction begins; thus, no more than five half-hour test sessions can be planned. Due to the homogeneity of the items developed from any one prototype, 150% of the number required for the final test were to be prepared for item tryout; the actual number of items prepared is given for each item type in Table 4.

As the test is to be developed for use with children entering first grade, children who will have had very little, if any, experience with testing, its format should be as simple as possible and should permit the teacher to observe the progress of all members of the group being tested. Although placeholders have frequently been used in tests developed for children in the primary grades, they are such small pictures that their effectiveness depends on the children's attention to directions and recognition of the objects. Large blocks of color could, however, be discriminated by a test administrator from any point in the room. Five colors of paper were used to print the tests, with consecutive pages printed on a different color and only one test item printed per page. This method not only permits the test administrator to tell at a glance whether all members of the class are ready for the item she is about to read but also may encourage the children to correct themselves if they turn two pages at once and notice the color they are looking for on the back of the pages turned.

Test directions are given in Appendix A; a copy of the tests appears in Appendix B. Although the items were not numbered in the test booklets for tryouts, numbers have been written on the copies of the booklets in the Appendix.

ITEM TRYOUTS

One child, the son of a staff member, participated in an informal pilot test of the booklets. This kindergartner completed five booklets in approximately 35 minutes, slightly more than the time estimated for one book, and made only five errors. From pilot testing it was possible to identify instructions that needed to be reworded prior to item tryouts and to reduce the estimate of the time required for testing. Of the five errors made in pilot testing, two resulted from the items themselves; in both items rowboats were drawn in perspective so that it was difficult for the child to determine whether there were length differences and where those differences were. The remaining three errors occurred near the end of testing; in forming sets with a specified number of elements the child carefully counted the correct number of objects and then quickly circled fewer objects without checking his work. His familiarity with numbers and numer concepts as well as his quick responses indicate that the child who participated in pilot testing was not typical.

From one of the Madison, Wisconsin, public schools cooperating with the R & D Center in research and development activities, fifteen children from an instructional unit of over 100 children were selected by the teacher to participate in item tryouts; intact classes were tested at Kegonsa and Yahara Schools, Stoughton, Wisconsin. Each test was administered to approximately 40 children; the number of subjects taking each test is given in Table 5. All tests were administered by the experimenter; in testing class-size groups in Stoughton she was aided by a member of the Project staff.

Table 5

Number of Subjects Taking Each Test

Source	Test Number						
	I	II	III	IV	V	VI	VII
Madison Schools--DMP Trained	15	15	14	13	14	14	13
Stoughton Schools--Other	30	30	23	25	24	24	24
TOTAL	45	45	37	38	38	38	37

All of the children from Madison had received instruction through Topic 6 of DMP Arithmetic Book I and, according to the teacher, represented a wide range of ability. Two ability groups were defined by the teacher and all seven tests were administered to the two groups separately during the week of May 18. Testing was carried out in one of two adjoining kindergarten rooms while large group activities were conducted in the other room. Children in the upper ability group generally responded during or immediately after the first reading and became impatient during the second reading of instructions; those in the lower ability group generally responded during or after the second reading, although their frequent exclamations between readings seemed to indicate that lack of attention contributed to their slower performance.

On May 21, 1970, the tests were administered to seven kindergarten classes in Stoughton. The Greater Cleveland Mathematics Program (Educational Research Council, 1962) is used in those classes; content includes counting and recognition of the numerals 1-10, ordering, one-to-one correspondence, and readiness for addition and subtraction. Test administrators were introduced as teachers from the university who had some booklets or games for the children to do. In all classes the regular teacher was not in the room during testing; in four classes the practice teacher remained in the room during test administration.

In all item tryouts the children, with a few exceptions, were separated approximately two feet or more.

SCORING

Originally all items were designed to be scored dichotomously; however, the children's responses indicated the need for a "half-right" rating on Tests III and IV. For example, when the children were asked to equalize lengths, they were directed to add or take away alternately. Many children compared the objects to be equalized (half right) but did not indicate any differences in equalizing by adding and equalizing by taking away (see Figure 2). Although these children did not show an understanding of the equalization process, their demonstrated comprehension of the comparison process will enable them to equalize with little additional instruction in contrast to the children whose answers indicated no ability to compare objects and determine how they differ in length.

Children's responses to items in Tests VI and VII were not always identical to those anticipated. In responding to the first items, children were to draw circles around a set containing a specified number of objects; although most children did circle the sets, some showed their knowledge of the numbers by drawing a continuous line through the specified number of objects or by numbering objects to indicate the quantity specified. These responses were considered evidence that the child had met objectives 15.3 and 16.3. The second part of Tests

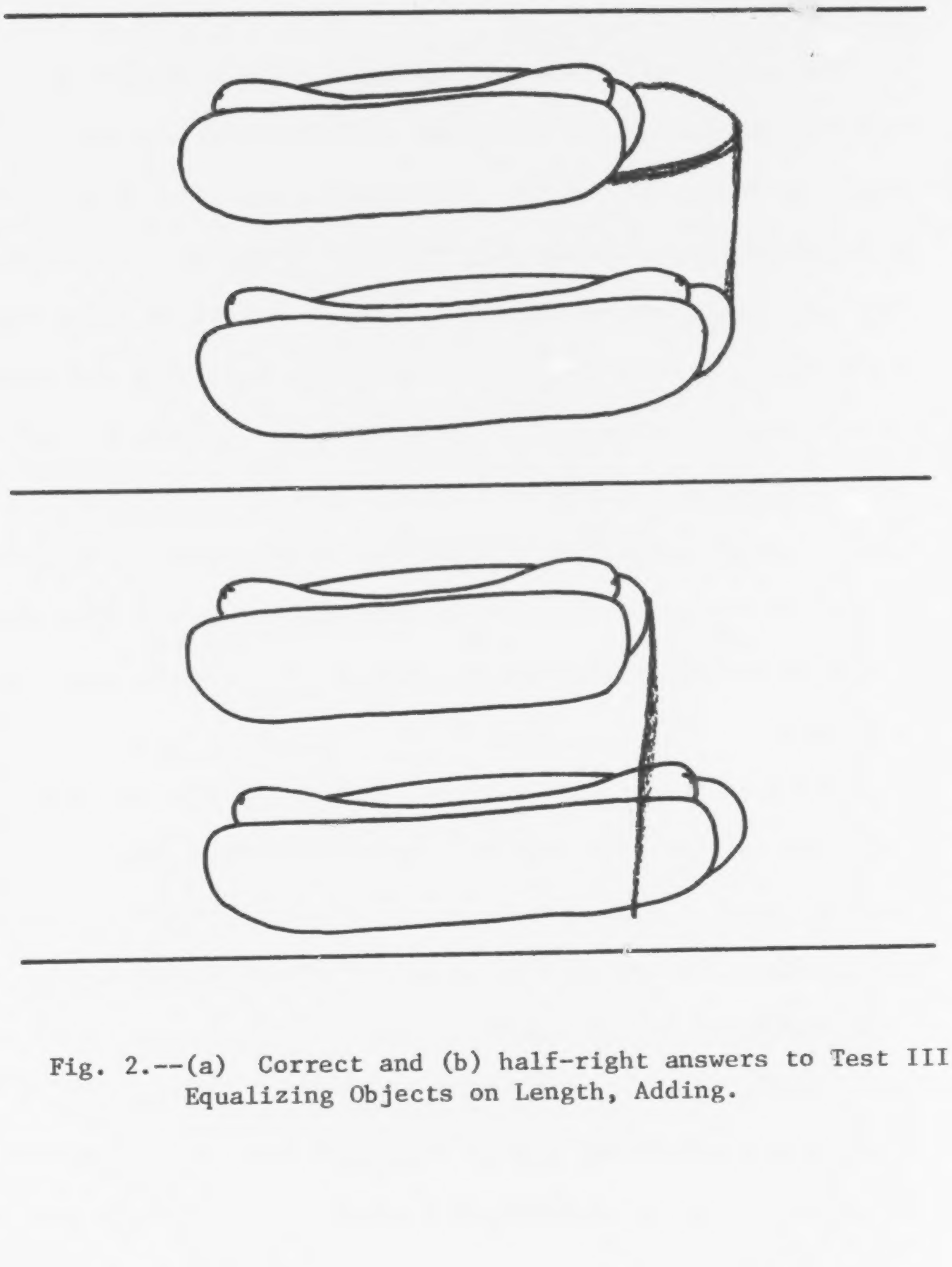


Fig. 2.--(a) Correct and (b) half-right answers to Test III, Equalizing Objects on Length, Adding.

VI and VII consisted of sets for which the children were asked to select the correct numeral from all the numerals 0-10. A series of circles around the numerals from 1 to the correct numeral was considered a correct response.

CONTENT VALIDITY

Each of the four first-grade teachers who had used Arithmetic Book I during the 1969-70 school year and all members of the Project staff were asked to respond to questions about a sample test. The letter sent to teachers, the memo circulated to the Project staff, and the sample test are in Appendix C.

DATA ANALYSIS

Item analyses were performed to secure item difficulty level, item-criterion (total test score) correlation, and item discrimination data; internal consistency reliability estimates were computed using the Hoyt ANOVA procedure. Subtest intercorrelations were calculated to determine which subtests differed from one another sufficiently to be reported as separate scores.

III

RESULTS

FEASIBILITY OF THE INSTRUMENT

Testing was conducted in class-size groups of 23 to 30 children as well as in ability groups of 6 to 8 children. In all item tryouts the children, with a few exceptions, were separated approximately two feet or more. Their curiosity about their classmates' work was no greater when testing was conducted in class-size groups rather than in smaller ability groups. Those children who were confident of their answers, however, would mark an answer before they tried to check their friends' work in contrast to the children who looked around immediately after the question was read. Printing sequential pages of the test on different colors of paper was an effective method of assisting the test administrator to determine whether all children had their booklets open to the correct page.

Children in Madison said they enjoyed the games. In Stoughton children generally reported that the tests were fun and easy, although one child whispered to the test administrator that he thought the games (Test III) were hard. In the Stoughton class that answered Test I, a girl offered the opinion that the test was more fun than mathematics. The most convincing evidence that the tests were appealing to children was given by a Stoughton boy who customarily sobs hysterically during any testing and had refused to leave the room with the teacher. About halfway through the booklet (Test II) the child began smiling and marking answers.

CONTENT VALIDITY

Sample items were distributed to the four first-grade teachers who had used DMP Arithmetic Book I in their classes during the 1969-70 school year and to seven members of the Analysis of Mathematics Instruction Project staff. The set of sample items is given in Appendix C along with the letter to teachers and the staff response sheet. One teacher asked whether items on the direction or orientation of numerals were to be added; no other questions about topics not included were raised.

All of the teachers and five staff members answered Yes to all questions about correct answers to items like the samples being satisfactory evidence that children had attained particular objectives. The two staff members not reporting that they would accept the item types objected to only one item type each. One staff member felt that the two stages in comparison of objects were not clearly tested; he recommended items specifically designed to test children's differentiation between equal and not equal as well as those included which tested differentiation between equal and longer or equal and shorter. The question for test construction was whether any additional information would be gained as a result of including such items. Although the question is testable, it would seem that children able to make the distinction between equal and unequal would mark both of the objects equal in length and only one of the objects unequal in length; thus, the information desired might be available from the test as it is now constructed. The other staff member objected to the arrangement of elements in the sets to be equalized on the property of numerousness; he felt that setting up a one-to-one correspondence for the children permitted them to equalize on the property of length rather than numerousness. More to the point, however, is the question raised by another staff member about the difference in difficulty between items in the Equalizing Sets category and those in the Ordering Sets category which do not have the one-to-one correspondence established. He felt that there are many intermediate levels of difficulty that should be represented in the test. In the Equalizing Test children must make a mark for each object they would add or take away, whereas in the Ordering Test they must only determine the set that is next largest or smallest in number; equal spacing in the Equalizing test was provided to eliminate the need for children to set up one-to-one correspondences, an operation that would be time consuming over many items, and to avoid obtaining test scores that could show either that children did not understand the equalizing process or that they were still unable to separate the numerousness of a set from the overall space it occupies, i.e., did not demonstrate what Piaget labeled conservation of numerousness. Two questions might be answered by comparison of scores from the tests with variations in spacing: first, what is the effect on test and item characteristics of providing children with potential problems related to conservation of numerousness? and second, do the activities provided in DMP assist children to differentiate the numerousness of a set from its other properties at an earlier age than do instructional activities provided by other programs? For initial item tryouts, the test was constructed to avoid consideration of either of these questions.

A question about the necessity of including all of the numerals from 0 to 10 in order in the last section of the Numbers and Numerals Test was also raised by a staff member. Many variations in the number and order of the numerals provided with each item might be tested; however, the items were typically answered correctly by 60-80% of the children in the tryout sample and, as counting from the first numeral given, zero, would result in an incorrect answer, would probably not become a great deal more difficult if fewer numerals were provided.

Teachers reviewing the sample test suggested a number of changes in directions, some of which were made before pilot testing. In Equalizing Objects the directions were changed from "Color to show . . ." to "Use your crayon to show . . .". In Comparing, children were given the directions for responding to equal objects and sets first, then unequal, with both directions beginning with if; original directions had indicated which of the different objects or sets were to be marked and then included instructions for the case of equal objects or sets. Too late for pilot testing was the suggestion that instructions for Equalizing Sets might be made more clear by indicating that the X was to go on, rather than over, the objects taken away and that the problem should be stated first (Make the set of scissors and the set of crayons equal in number) and the method second (by making a circle for each object you must add to the smaller set). Teachers generally commented that the sets in Tests I and II should be separated; lines broader than those used in drawing the pictures could be used to draw a loop around each set. Another typical comment was that the objects were too small for the children to mark around or over accurately using blunt crayons; during item tryouts children frequently began their crayon marks between the two nearest objects and drew carefully only as they began to mark their responses. If all or most of the objects were quite small, the children would probably tire very quickly and begin to make frequent errors; the high mean scores reported in the following section would appear to indicate that the amount of careful work required is not sufficient to affect children's overall test performance. It should be noted that increasing the size of objects in items that picture many objects would necessitate a change in the test format.

One teacher's suggestion that sets to be equalized should consist of objects the children can draw--circles, X's, triangles, or squares for example--is excellent; providing an entire test of such objects would raise considerable difficulties, not only in devising the required number of different items but also in convincing children that their drawings did not have to be accurate reproductions to be considered good. Children's preference for objects they can draw themselves was indicated in the sample item for Equalizing Sets, Adding, by the number of glasses drawn over the circle which had been inserted to represent an added glass. As the Project is planning to provide teachers with exercises to be administered individually to children whose performance on the group test is of unacceptable or uneven quality, staff should consider recommending sets that the teacher constructs for the child to equalize by drawing additional objects.

ITEM ANALYSIS

Summary data for all seven tests are given in Table 6. Although means and standard deviations were reported separately for the two groups of students tested, other test and item statistics were calculated using combined results from all students tested. For four of the seven tests, acceptable internal consistency reliability estimates ($r > .90$) were obtained. Of the three tests with low reliabilities, two were the initial classifying and comparing tests; these two tests were very easy (mean scores of 20.1333 and 19.9556 for 24-item tests). In Test V, Ordering, 8 of the 23 items were of the easy classifying and comparing type; the mean score for these 8 items was 6.9843. Scoring of Tests III, Equalizing Objects, and IV, Equalizing Sets, with a comparison as well as equalization marked correct did raise the average scores, although the increases were less than had been expected. Variability overall changed little; however, DMP-trained subjects showed a striking decrease in variability from the first to the second scoring (Test III, $s^2 = 49.6946$ and 21.6383; Test IV, $s^2 = 24.7940$ and 12.9955). Changes in directions and in sample items as described in Part IV were recommended to attempt obtaining a more accurate indication of the children's knowledge without giving credit for half-right answers. Recommendations for items to be retained in the final form of these two tests were based on results from scoring only equalizations correct; in this way, decisions about the items were based

Table 6

Summary of Mean Scores, Reliabilities, and Related Statistics

for All Seven Tests

Test title	Statistic	DMP-trained subjects	Other subjects	Total	No. of items	r	Standard error
Test I, Classifying and Comparing Objects and Sets I	Mean	20.3125	19.9333	20.1333	24	.6346	1.4252
	Std. dev.	2.2835	2.4487	2.4083			
	Range	17 - 24	14 - 23	14 - 24			
Test II, Classifying and Comparing Objects and Sets II	Mean	20.7333	19.5666	19.9556	24	.6786	1.6135
	Std. dev.	2.6949	2.8833	2.9073			
	Range	14 - 24	12 - 24	12 - 24			
Test III, Equalizing Objects on Length (equalization only correct)	Mean	14.1428	11.6956	12.6216	22	.9178	1.7967
	Std. dev.	7.0494	5.6448	6.4133			
	Range	1 - 21	1 - 21	1 - 21			
Test III, Equalizing Objects (equalization & comparison correct)	Mean	19.0714	16.6521	17.5676	22	.9383	1.3166
	Std. dev.	4.6516	5.5374	5.4239			
	Range	3 - 21	2 - 21	2 - 21			
Test IV, Equalizing Sets on Numerousness (equalization only correct)	Mean	16.2307	11.7600	13.3158	22	.9234	1.7219
	Std. dev.	4.9793	6.3515	6.3676			
	Range	7 - 22	0 - 22	0 - 22			
Test IV, Equalizing Sets (equalization and comparison correct)	Mean	18.9230	16.2800	17.1842	22	.9488	1.3937
	Std. dev.	3.6049	7.0513	6.3968			
	Range	8 - 22	0 - 22	0 - 22			

(Continued →)

Table 6 (cont'd.)

Test title	Statistic	DMP-trained subjects	Other subjects	Total	No. of items	r	Standard error
Test V, Ordering More Than Two Objects and Sets	Mean	16.4285	14.5833	15.2632	23	.7368	1.8691
	Std. dev.	4.8509	2.5318	3.7249			
	Range	6 - 22	10 - 21	6 - 22			
Test VI, Numbers and Numerals I	Mean	22.4285	22.5833	22.5263	28	.9132	1.6923
	Std. dev.	6.5544	5.2593	5.8482			
	Range	4 - 28	4 - 28	4 - 28			
Test VII, Numbers and Numerals II	Mean	22.0769	20.6250	21.1351	28	.9147	1.7978
	Std. dev.	6.9775	5.6407	6.2679			
	Range	3 - 28	4 - 27	3 - 28			

on data obtained when items were scored as they would be in the final test.

In the following sections, recommendations for revisions in the test and for selection of final forms of the tests are presented. Recommendations for item selection were based on the criteria summarized by Harris (1968). Difficulty level, item test correlations, and discrimination indices for each item are given in Appendix D. In all cases the statistics are those from analysis of all items in a single test; for each choice of the Classifying items in Tests I and II, the statistics are those from analysis of Classifying Objects and Classifying Sets alone.

After selection of items had been completed, conservative estimates of test reliabilities were calculated using the Spearman-Brown formula (Lord and Novick, 1968, p. 112):

$$\hat{r} = \frac{kr}{[1 + (k - 1)r]}$$

where k indicates the factor by which the test length is altered and r, the original internal-consistency reliability estimate. For the pairs of Tests I and II and Tests VI and VII, reliability estimates were calculated separately for each test as though only items like those in that test would be included in the lengthened instrument. Results are presented in Table 7. In calculating reliability estimates the assumption was made that the tests would be composed of items identical in type to those in the initial tryout.

Tests I and II, Classifying and Comparing Objects and Sets

As stated previously, these two tests were very easy. Statistics for both total tests and for subscales are summarized in Table 8. Of the 48 items tried out, 21 were to be selected for a final test, 15 of the classifying type and 6 of the comparing type. For the four items in both tests

Table 7

Estimated Reliabilities for Tests

of Different Length

Test No.	No. of items tried out	No. of items to be retained[a]	No. of items in final test	Orig. r	Est. r
I	24	10	21	.6346	.6245
II	24	11	21	.6786	.6623
III	22		14	.9178	.8764
IV	22		14	.9234	.8858
V	23		16	.7368	.6389
VI	28	17	35	.9132	.9291
VII	28	18	35	.9147	.9304

[a] Indicates number different from that for final test.

Table 8

Summary of Statistics for Tests I and II,

Classifying and Comparing Objects and Sets,

Total Test and Subscales

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Test I, Classifying and Comparing Objects and Sets I						
Comparing objects on length	6	3-6	4.2889	.7869	.0666	.6940
Comparing sets on numerousness	6	1-6	4.8222	1.7350	.6585	.7483
Comparing objects and sets	12	5-12	9.111	1.7350	.5572	1.1054
Classifying objects	6	4-6	5.8000	.5477	.4283	.3781
Classifying sets	6	2-6	5.2222	.8762	.2511	.6922
Classifying and comparing objects	12	8-12	10.0889	.8744	-.0187	.8450
Classifying and comparing sets	12	6-12	10.0444	1.8581	.6185	1.0987
Total test	24	14-24	20.1333	2.4083	.6346	1.4252
Test II, Classifying and Comparing Objects and Sets II						
Comparing objects on length	6	2-6	4.3778	1.1734	.2685	.9161
Comparing sets on numerousness	6	0-6	4.5333	1.7658	.7845	.9161

(Continued)

Table 8 (cont'd.)

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Comparing objects and sets	12	3-12	8.9111	2.3435	.6641	1.3003
Classifying objects	6	4-6	5.4000	.6537	-.1645	.6439
Classifying sets	6	4-6	5.6444	.5290	-.1776	.5240
Classifying and comparing objects	12	6-12	9.7778	1.5505	.3855	1.1637
Classifying and comparing sets	12	6-12	10.1778	1.9690	.6967	1.0382
Total test	24	12-24	19.9556	2.9073	.6786	1.6135

that call for selection of one of two objects or sets (Items 1, 2, 9, and 10) there were only three errors out of 180 responses. These items might be left in the test but should not be scored. One item should be omitted from Classifying Objects because it was too easy, had a low correlation with total test score, and discriminated poorly at the low end of the score range (Item II-18: difficulty = .9778, $r = .1380$, $X_{50} = -14.5661$, $\beta = .1393$). Item II-20 showed poor results primarily because of the teakettle which was not considered an object used for cooking; of the ten children who did not mark the teakettle, nine marked the rest of the item correctly. The teakettle should be omitted from a revised test. Similar results were obtained for Classifying Sets; two items should be dropped because of their difficulty and item-test correlation (Item II-21: difficulty = .9778, $r = .2824$, $X_{50} = -7.1175$, $\beta = .2944$; Item II-23: difficulty = .9333, $r = .0512$, $X_{50} = -29.3009$, $\beta = .0513$) and the set of tents should be omitted from Item I-24.

Items from Comparing were poorer overall than those from Classifying. Items were selected for the final test so that two items were retained for each relationship (longer or larger in number, shorter or smaller in number, equal in length or number) for objects and sets. The resulting instrument is not of good quality; recommendations are made in Part IV. Items I-6 and I-8 were omitted from Comparing Objects because all children answered them correctly. Items I-3 and I-5 were also too easy (difficulty = .8889) and they showed low correlations with total test score, .3248 and .4222 respectively. To retain two items that pictured objects equal in length, it would be necessary to keep an item with poor characteristics (II-7: r = .2924, β = .3057) as two showed negative correlations with total test score (I-7, r = -.0725; II-4, r = -.0379). The pictures used in items that had objects of equal length were, in three out of four cases, an unfortunate choice; children seeing rowboats, tables, and books that were drawn in perspective responded on the basis of nearest edges rather than overall size as shown by lines some of them drew to show the comparison. Other objects or the same objects not drawn in perspective should be pictured in any new items tried out.

Items for Comparing Sets were of higher quality than those for Comparing Objects as can be seen from the item-test correlations and values of some items omitted. The following obviously poor items were initially dropped: I-14, difficulty = .9333, r = .4567, β = .5134; I-16, difficulty = .8444, r = .4610, β = .5195; II-12, difficulty = .5778, r = .3937, β = .4283. One additional item for each of the numerousness relationships was omitted, II-11, II-13, and II-15.

Test III, Equalizing Objects

Summary statistics for both scorings of the total test and the subscales are presented in Table 9. Of the 22 items tried out, 8 were to be omitted from the final form of the test. Equal numbers of add and take away items were tried out and were recommended for the final test. Decisions were based on the data obtained when only equalizations were scored correct.

Table 9

Summary of Statistics for Test III,

Equalizing Objects on Length,

Total Test and Subscales

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Only correct equalization scored correct						
Equalizing objects by adding	11	0-11	5.4865	4.2598	.9449	.9534
Equalizing objects by taking away	11	0-11	7.1351	4.4793	.9608	.8456
Total test	22	1-21	12.6216	614.33	.9178	1.7967
Correct equalization and comparison scored correct						
Equalizing objects by adding	11	0-11	8.2432	2.9383	.8873	.9406
Equalizing objects by taking away	11	2-11	9.3243	2.6146	.8746	.8828
Total test	22	2-21	17.5676	5.4239	.9383	1.3166

Rowboats were so difficult to equalize that Item 2 picturing them showed a positive correlation between the incorrect answer and total test score. Five other items in the test had β values < 1; four of them (Items 3, 4, 5, and 19) should be omitted, the fifth, Item 6, being retained as the best of the four take away items in this group. The third add item to be omitted, along with Items 2 and 5, should be Item 7 which had the smallest β value for the wrong response (-.4670) of the remaining add items. In responding to this item, many children tried to show lengthening the chair legs instead of the chair back; perhaps lengthening wood is reasonable to the children and the item would have been satisfactory if wooden rather than upholstered chairs had been pictured.

One of the remaining take away items should probably be tried out in a slightly different form. Item 18 pictured two kites flying at different heights from the same baseline; taking away from the top of the kite, as necessary for a correct response, was unnatural and caused the child in pilot tryout and some children in other testing situations to spend time trying to figure out how to show the kite string pulled in before they answered correctly. This item should be retested with kites at the same level and strings different lengths.

Test IV, Equalizing Sets

In Table 10, the summary statistics for this total test and subtests are presented. As in Test III, data from the scoring that credited only equalizations as correct responses were used in determining which

Table 10

Summary of Statistics for Test IV,
Equalizing Sets on Numerousness,
Total Test and Subscales

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Only correct equalization scored correct						
Equalizing sets/ difference of one	5	0-5	3.2105	1.7110	.7793	.7190
Equalizing sets/ difference of two	7	0-7	4.4211	2.0616	.7571	.9407
Equalizing sets/ difference of three	5	0-5	2.8421	1.6850	.7653	.7302
Equalizing sets/ difference of four	5	0-5	2.8421	1.5[illegible]14	.6723	.7944
Equalizing sets by adding	11	0-11	5.0526	4.0600	.9171	1.1147
Equalizing sets by taking away	11	0-11	8.2632	3.0991	.8633	1.0926
Total test	22	0-22	13.3158	613767	.9234	1.7219
Correct equalization and comparison scored correct						
Equalizing sets/ difference of one	5	0-5	4.0526	1.6099	.8747	.5097
Equalizing sets/ difference of two	7	0-7	5.7105	2.0389	.8655	.6924

(Continued)

Table 10 (Cont'd.)

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Equalizing sets/ difference of three	5	0-5	3.6579	1.5644	.7650	.6783
Equalizing sets/ difference of four	5	0-5	3.7632	1.6347	.8058	..6443
Equalizing sets by adding	11	0-11	8.2632	4.4145	.9075	.9899
Equalizing sets by taking away	11	0-11	8.9211	3.0258	.8945	.9371
Total test	22	0-22	17.1842	6.3068	.9488	1.3937

items would be omitted in a final form; 8 of the 22 tested items were to be omitted. For five of the items (2, 4, 9, 15, and 19) β values were less than 1; four of these items should be omitted, although one, Item 19 preferably, will have to be retained to keep the number of add items equal to the number of take away items. Item 19 had better characteristics than the other take away items under consideration (difficulty = .6316, r = .6732, β = .9105). As all of the remaining add items showed high correlations with total test score, β values for the correct response greater than 1, and similar discrimination points ($.0681 \leq X_{50} \leq .3178$), the two recommended for exclusion were those with the smallest negative β values for the incorrect response, Items 7 (β wrong = -.6868) and 13 (β wrong = -.6308). Equalizing sets with a difference of one and equalizing those with a difference of two appeared to be of similar difficulty but easier than equalizing sets with differences of three or four. The recommended selection would reduce each of the two groups by three items.

Test V, Ordering More Than Two Objects or Sets

In Table 11 statistics summarizing the results of administration of Test V are presented. The total test of 23 items was to be reduced by 7 items in agreement with the revised test development guide (Table 4); 4 items were omitted from the 8 related to identifying longest, smallest in number, etc., with the remaining 3 omitted from the 15 items testing children's ability to order the next object or set.

Items 2 and 3, identification of the tallest object, did not differ and were good but easy items (difficulty = .9474, r = .8895,

Table 11

Summary of Statistics for Test V,

Ordering More Than Two Objects or Sets,

Total Test and Subscales

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Identifying longest/ shortest	4	2-4	3.8421	.4946	.4884	.3064
Ordering next longest/shortest	8	0-7	4.4474	2.0494	.6476	1.1380
Identifying largest/ smallest in number	4	1-4	3.3421	.9087	.4915	.5612
Ordering next largest/smallest	7	0-7	3.6316	1.7151	.5320	1.0863
Ordering objects on length	12	2-11	8.2895	2.2167	.6524	1.2512
Ordering sets on numerousness	11	4-11	6.9737	2.0987	.5702	1.3119
Total test	23	6-22	15.2632	3.7249	.7368	1.8691

X_{50} = -1.8211, β = 1.9468). Items 1 and 4, identification of shortest object, were also easy, but neither item discriminated well at a reasonable level or correlated with total test score. These results may reflect children's knowledge of longest and shortest; however, since objects were drawn horizontally for the selection of shortest and vertically for the selection of longest, the results may indicate children's preference for making comparisons vertically. The items should be tried out again with changes in wording. Without tryouts, items selected to test knowledge of longest and shortest would be either 2 or 3 and 4.

Items testing knowledge of largest in number each had four sets and were easier but had lower item-test correlations than those testing knowledge of smallest in number. Again, if additional items were tried out, more information could be gained not only about largest/smallest but also about the arrangement of objects in sets which was questioned by staff members reviewing sample items. The two items to be selected without additional item tryouts are 15 and 16 as each discriminated better and had a higher item-test correlation than the other choice in its category.

Items 6 and 10, testing children's ability to select the next object when one or two are in order by length, were of markedly poorer quality than the six others in that group (r = .1266 and .4373, β = .1276 and .4863 respectively; nearest r = .5735, nearest β = .7000) and should be omitted. In both of these items the correct choice was the same height as the object already in place; although a place was provided for all objects to be put in order, children seemed to respond to the request for the next object by length rather than next of all those provided. A change in directions to emphasize that all objects do have a place to be put in order could be tested. The inclusion of sets to be ordered that were equal in number to those ordered did not lead to similarly poor item characteristics. Items 20 and 22 both had item-test correlations near .40 and neither discriminated well (β = .4696 and .4400 respectively); Item 22 should be omitted to keep the best 6 items.

Tests VI and VII, Numbers and Numerals

Summary statistics for the total tests and for subscales composed of items developed from one prototype are given in Table 12. From the 56 items tried out in both of these tests, 21 were to be omitted to leave one 35-item test. Three methods of selecting items were considered. First, the best 35 could be selected on the basis of difficulty, item-test correlation, and discrimination indices; although sound from a theoretical point of view, this method does not allow for any later determination of differences among the item types. It should be noted that the internal consistency reliability estimates over .90 obtained from tryouts of the tests indicate little or no difference among the abilities measured by the different item types; however, until additional evidence to support the similarity of the skills is gathered, the more conservative interpretation that the children participating in item tryouts had mastered all skills equally well is urged. From

Table 12

Summary of Statistics for Tests VI and VII, Numbers and Numerals, Total Test and Subscales

Subtest title	No. of items	Score range	Mean score	Standard deviation	r	Standard error
Test VI, Numbers and Numerals I						
Selecting sets/ number given	8	1-8	6.1842	1.6903	.6197	.9751
Selecting sets/ numeral given	8	0-8	6.2632	1.9957	.7906	.8542
Selecting sets/number or numeral given	16	1-16	12.4474	3.3828	.8303	1.3292
Selecting numeral to represent sets	12	2-12	10.0789	2.8700	.8857	.9292
Total test	28	4-28	22.5263	5.8482	.9132	1.6923
Test VII, Numbers and Numerals II						
Selecting sets/ number given	8	0-8	6.4054	1.7394	.6772	.9244
Selecting sets/ numeral given	8	0-8	5.5946	2.6818	.9016	.7868
Selecting sets/number or numeral given	16	0-16	12.0000	3.9791	.8814	1.3271
Selecting numeral to represent sets	12	2-12	9.1351	2.7301	.8151	1.1239
Total test	28	3-28	21.1351	6.2679	.9147	1.7978

this interpretation, the second method of selecting items evolved, selecting the best 10 or 15 items within each category as specified in the test development guide. Selecting the best items from each category with the restriction that the eleven numerals 0-10 be represented equally in each category was the third method. Its rejection was based on the fact that the resulting test would be of poorer quality; if the final test is to include equal weighting of the numerals, additional items should be tried out to determine the effects of the pictures used on item characteristics before an adequate test can be constructed. In many cases difficulty seemed to relate to the picture rather than the numeral (e.g., in showing a bus eight windows long, children had to omit one of the wheels; one child showed his distaste for the unrealistic task by drawing two circles, each around eight windows, so that both wheels appeared to stay with the bus), although only further testing of the same pictures with different numbers and numerals will clarify the source of difficulty.

From the first 8 items in both tests, five were initially omitted because of their low correlations with total test score ($.1948 \leq r \leq .5965$) and little discriminating power ($.1986 \leq \beta \leq .7431$); these items were VI-3, VI-4, VI-5, VII-4, and VII-5. The sixth item recommended for omission was VI-8 which had, in addition to the lowest r and β value of remaining items, a low item-test correlation for its difficulty level (difficulty = .9474, r = .6724). Six items from the second eight of both tests with item-test correlations less than .80 were omitted to leave 10 items written from Item Type 6 (Items VI-9, VI-13, VI-15, VII-9, VII-11, and VII-12 omitted). Item-test correlations of .8524 and .8681 for Items VI-10 and VII-10 were the only remaining r values less than 1 in this group. In the final group of items in Numbers and Numerals, six items (VI-21, VI-27, VII-19, VII-22, VII-26, and VII-28) had item-test correlations less than .70 and should be omitted first. Four items correlated only slightly better with the total test ($.7003 \leq r \leq .7606$) and had similar β values ($.9811 \leq \beta \leq 1.1715$). Of these, Item VI-22 should be omitted because it discriminated at a lower level than the other three, $X_{50} = -1.5317$. Items VI-17 and VII-18 were much easier, difficulty = .6842 and .7297 respectively, than Item VII-20, difficulty = .3514, and might also be omitted. For the remaining 15 items, item-test correlations ranged from .7003 to 1.5341, the six calculated X_{50}'s from -2.0976 to .5450, and β values calculated from .9811 to 4.3881.

IV

RECOMMENDATIONS

In this Part, recommendations not specifically related to particular items administered are made, although some suggestions relate to specific tests.

RECOMMENDATIONS FOR SPECIFIC TESTS

Tests I and II, Classifying and Comparing Objects and Sets

Reading time for the length and numerousness comparisons in Tests I and II is extensive and differs greatly from that for the remaining items in those tests. Items like those described below could be tried out; such items more nearly follow the pattern of others in the same subtest as well as the items tried out with those for Ordering that should be part of this test. As in the item prototypes given earlier, the example is written for objects only for ease in reading.

> Child sees pairs of objects. Each pair in a box or otherwise set off.
> Teacher reads: Put an X on the box that shows two objects equal in length. OR
> Put an X on the box that shows the name of object 1 shorter (longer) than the name of object 2.

Tests III and IV, Equalizing Objects and Sets

Observation of the test administration led to the conclusion that directions for Equalizing are difficult enough that the scores may not accurately indicate children's understanding of the process. This conclusion is supported by the children's responses; many of them noted only one method of equalizing from the sample items and used only that method for all items. Separation of the add and take away items would probably eliminate most of the difficulty. This solution would, from one point of view, merely create an entire subtest of half-right responses; it would not be possible to determine whether children understood the equalization process or drew a line as one was drawn in the sample item. For Equalizing Objects a change in the marked sample items and the addition of samples that the class worked through together should be considered. On the marked samples, marks should extend over both objects at the point at which equalization is to end. That is, the line at which addition ends should run along the end of the longer object as well as beyond the shorter, and the line at which subtraction ends should run along the end of the shorter object as well as through the longer. These lines would appear in addition to the coloring now shown on sample items. Following each marked sample, the children should equalize a pair of objects so that they are more familiar with the responses desired before the test begins. Making these changes would probably result in a reduction of the high reliabilities obtained from tryout of Test III but would provide information about children's ability to equalize objects that was less clouded with their ability to listen and remember directions.

Samples for Equalizing Sets should have similar changes. In the marked samples one-to-one matching should be indicated by lines. The marked sample for adding would include lines going to the circles representing added objects; lines would not be drawn from the objects to be taken away. Children should be instructed to set up the correspondence in the samples they work through together. Again, making these changes would lower the obtained reliabilities.

As the sample for Equalizing Sets, Adding, is now marked, the circle is so nearly a solid line and so close in size to the glasses it appears next to that its insertion as a sample crayon mark is not immediately apparent; the circle should be enlarged and more roughly drawn. Directions for Equalizing Sets, Taking Away, should be changed from "Put an X on . . . " to "Cross out . . . ."

Test V, Ordering More Than Two Objects or Sets

As stated previously, the items in Ordering that require only identification of one of the objects on the page should be administered as part of Classifying. Teachers suggested that the ordered objects and sets be more clearly differentiated from the ones to be ordered; a double line might be used to divide the page and a broad line added near the lines that indicate locations for ordered objects and sets.

RECOMMENDATIONS FOR THE ENTIRE BATTERY

Mechanical

It is not necessary for the test administrator to read all directions for Ordering or for the last two sections of Numbers and Numerals. During item tryouts some children worked ahead, particularly on the last section of Numbers and Numerals. The effects on results of providing tests as independent work for the children should be investigated; a study could readily be conducted in an instructional unit.

Even in class-size groups children did not seem to have difficulty keeping their places in the test booklets. Scoring was more time consuming than it would have been if more than one item had been presented on each page. A study of the practicality of two, three, or four items per page, with testing groups varying in size from approximately 8 to 25, would provide information that either would permit construction of tests for kindergartners that could be more efficiently scored or could guide test constructors away from developing instruments that give results with little meaning.

A test battery that could be administered in four sessions of approximately equal length would probably be the most desirable outcome from a teacher's point of view, requiring less time overall than more sessions in which the same number of items were administered. Items of Types 1, 2, and 4a might be combined into one 26-item test of children's skills in classifying and comparing. Items of the remaining types do not readily combine into tests of the desired length. The 35 items of Types 5-7 should not be administered in one session; children were eager to reach the end of the 28-item tests administered in tryouts. (If it were feasible to let children complete the test independently, a 28- or 35-item test might not be as dull a task for them.) Some thought should be given to revising the weights assigned to Types 5-7; reliability of a 25-item test constructed from Tests VI and VII as they were administered is estimated to be approximately .90. Two alternatives for compiling tests of Item types 3 and 4b are possible. First, all items of Type 3 could be combined into one 28-item test and the items written for Type 4a administered as one 16-item test. (If this alternative is adopted, the Ordering Test could be lengthened.) Because the children have more to do in responding to items of Type 3 than they do in responding to items of Types 5-7, the 28-item test would probably not, in this case, be as dull a task for them. Second, items of Types 3 and 4b could be divided into two 20-item tests, the division being made to put items relating to objects in one test and those relating to sets in another. Project staff deciding between alternatives should consider the relative importance of the ordering and equalizing processes for the next units of instruction as well as the characteristics of the instruments constructed under the two alternatives.

Technical

To provide full technical data about the test, reliability and validity studies should be conducted as early as possible in the coming school year. Recommendations in Part III and in the previous sections for alterations in the test could, if adopted by the Project staff, alter the instrument enough to change the internal consistency reliability reported. It is essential that the reliability be recalculated following any format and arrangement changes. After the entire battery has been administered to a large group of children, intercorrelations among the tests will have to be calculated to determine those that differ from one another sufficiently that subscores should be recommended for plotting on individual profiles.

As the test was developed for the purpose of differentiating between the children who have the background to learn new concepts and those who need review or remedial work before they can take full advantage of additional instruction, an estimate of the predictive validity of the instrument should be sought. An instructional unit of approximately 100 children would provide a suitable setting for the validity study. After administration of the battery at the beginning of the school year, tests would be returned to the R & D Center for scoring and mathematics instruction provided to all children for 3-5 weeks as though they had met the criteria for continuing instruction. At the end of the specified period of instruction, a measure of the children's learning, criterion measure, would be obtained. If the battery has predictive validity, the correlation between scores on the battery and scores on the criterion should be high. A primary difficulty in conducting the validity study will be locating an appropriate reliable criterion measure. Available instruments that relate directly to instructional objectives are individually administered observation schedules with undetermined reliability.

Through factor analysis of the subscores, a test of the hierarchy could be performed. If the hypothesized hierarchy is a valid description of the mathematical content, the variance-covariance matrix obtained should form a Simplex (Kaiser, 1962); the Simplex fitting can be checked by a program now available at the University of Wisconsin Computing Center. Confirmation of the hierarchy would provide support for the task analysis developmental procedure and would nearly eliminate review tests. Once children had shown mastery of process objectives that were elements of a hierarchy, they would not have to be retested when new instruction building on those elements was to be introduced; the children's continual application of objectives they had mastered would be assured.

A useful instrument could be constructed from the items developed for this study, although the results of additional item tryouts would make possible the construction of a greatly improved instrument.

REFERENCES

Anastasi, A. Psychological testing. (3rd ed.) New York: Macmillan, 1968.

Bloom, B. S. Learning for mastery. In Evaluation comment. Los Angeles: Center for the Study of Evaluation of Instructional Programs, University of California, 1968.

Cronbach, L. J. Essentials of psychological testing. (2nd ed.) New York: Harper & Brothers, 1960.

DeVault, M. Vere, Osborn, R., & Forester, H. B. Discovering mathematics 4. Columbus, Ohio: Merrill, 1966.

Educational Research Council of Greater Cleveland. Greater Cleveland mathematics program: Teacher's guide for kindergarten. Chicago: Science Research Associates, 1962.

Eicholz, R. E., & O'Daffer, P. G. Teachers' edition to accompany Elementary School Mathematics Book 4. (2nd ed.) Menlo Park, Calif.: Addison-Wesley, 1968.

Gagne, R. M., Mayor, J. R., Garstens, H. L., & Paradise, N. E. Factors in acquiring knowledge of a mathematical test. Psychological Monographs, 1962, 76(7, Whole No. 526).

Gagne, R. M., & Paradise, N. E. Abilities and learning sets in knowledge acquisition. Psychological Monographs, 1961, 75(14, Whole No. 518).

Harris, M. L. Some methodological suggestions for construction of an objective measurement instrument. Technical Memo No. M-1968-2. (Rev.) Madison: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, 1968.

Hartung, M. L., Van Engen, H., Gibb E. G., Stochl. J. E., Knowles, L., & Walch, R. Seeing through arithmetic 4. Glenview, Ill.: Scott, Foresman, 1967.

Kaiser, H. F. Scaling a Simplex. Psychometrika, 1962, 27, 155-162.

Klausmeier, H. J., & Dresden, K. Teaching in the elementary school. (2nd ed.) New York: Harper & Brothers, 1962.

Klausmeier, H. J., Morrow, R. G., Walter, J. E., & Way, R. S. Individually guided education in the Multiunit School: Guidelines for implementation. Bulletin 9-168. Madison: Wisconsin Department of Public Instruction, 1968.

Lord, F. M., and Novick, M. R. Statistical theories of mental test scores. Reading, Mass.: Addison-Wesley, 1968.

Romberg, T. A., Fletcher, H. A., & Scott, J. A. A measurement approach to elementary mathematics instruction. *Working Paper No. 12.* Madison: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, 1968.

Romberg, T. A., Harvey, J. G., & McLeod, D. B. The task analysis for *Developing Mathematical Processes* Arithmetic Book I, Comparing and Equalizing Objects and Sets. *Working Paper, in press.* Madison: Wisconsin Research and Development Center for Cognitive Learning, University of Wisconsin, 1970.

Stake, R. E. Toward a technology for the evaluation of educational programs. *In Perspectives of curriculum evaluation.* American Educational Research Association Monograph Series on Curriculum Evaluation No. 1. Chicago: Rand McNally, 1967. Pp. 1-12.

Rinsland, H. D. *A basic vocabulary of elementary school children.* New York: Macmillan, 1945.

Thorndike, E. L., & Lorge, I. *The teacher's word book of 30,000 words.* New York: Bureau of Publications, Teachers College, Columbia University, 1944.

Tyler, R. W. Changing concepts of educational evaluation. In *Perspectives of curriculum evaluation.* American Educational Research Association Monograph Series on Curriculum Evaluation No. 1. Chicago: Rand McNally, 1967. Pp. 13-18.

APPENDIX A

Test Directions

GENERAL DIRECTIONS FOR ALL TEST SESSIONS

To insure meaningful results, you must follow the directions carefully. Read each question twice.

Do not read letters and numerals in the left margin of specific test directions to the children; these figures are for your reference only. Sample items are indicated by letters, while items that will be scored are identified by numerals.

In addition to this manual of directions, you will need a test booklet for each child, an extra booklet for demonstration, and a crayon for each child. If the children work at tables, you may either have them get crayons before they are seated or distribute the crayons yourself when you distribute the test booklets.

You may either ask the children to write their names on the booklets at the time of testing and complete the identifying information yourself later or enter all information on the front of the booklets prior to testing.

DIRECTIONS FOR TEST I, CLASSIFYING AND COMPARING OBJECTS AND SETS I

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon.

If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Look at the squares at the bottom of the booklet.

Hold up demonstration booklet and point to squares.

Then say:

One box has a picture of an apple in it, and the other has a picture of a pear. A big X has been marked on the apple. You will answer the games in this booklet by putting a big X on pictures. Show me how you will mark your answers. Put a big X over the pear.

Check to see that all children mark the pear. Then say:

Very good. Now open your books to the yellow page with a picture of cans on it.

Hold up demonstration booklet opened to the correct page and check to see that all children have that page. Then say:

Λ Look at the cans on this page. Find the can that is open and put your finger on it.

Check to see that children have the correct answer.

Then say:

That's right. The open can has a big X on it. You will do the rest of the games in this book by making a big X on the answer. Try to make an answer to each game. Do you have any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the green page. Put an X on the coat that has buttons.

2 Now turn to the pink page. Put an X on the tree that does NOT have leaves.

3 Now turn to the blue page. If the feet are equal in length, put an X on both feet. If they are not equal in length, put an X on the longer foot.

4 Now turn to the white page. If the tables are equal in length, put an X on both tables. If they are not equal in length, put an X on the longer table.

5 Now turn to the yellow page. If the houses are equal in height, put an X on both houses. If they are not equal in height, put an X on the shorter house.

6 Now turn to the green page. If the pencils are equal in length, put an X on both pencils. If they are not equal in length, put an X on the shorter pencil.

7 Now turn to the pink page. If the boats are equal in length, put an X on both boats. If they are not equal in length, put an X on the shorter boat.

8 Now turn to the blue page. If the sleds are equal in length, put an X on both sleds. If they are not equal in length, put an X on the longer sled.

9 Now turn to the white page. Put an X on the set of things you eat.

10 Now turn to the yellow page. Put an X on the set of things that do NOT play music.

11 Now turn to the green page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

12 Now turn to the pink page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

13 Now turn to the blue page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

14 Now turn to the white page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

15 Now turn to the yellow page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

16 Now turn to the green page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

17 Now turn to the pink page. Put an X on all of the things that are soft.

18 Now turn to the blue page. Put an X on all of the children.

19 Now turn to the white page. Put an X on all of the things that do NOT fly.

20 Now turn to the yellow page. Put an X on all of the things people read.

21 Now turn to the green page. Put an X on all of the sets of things you use in winter.

22 Now turn to the pink page. Put an X on all of the sets of things you do NOT drink from.

23 Now turn to the blue page. Put an X on all of the sets of things you could lift by yourself. These are pictures of real objects, not toys.

24 Now turn to the white page. Put an X on all of the sets of things people live in.

DIRECTIONS FOR TEST II, CLASSIFYING AND COMPARING OBJECTS AND SETS II

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Look at the squares at the bottom of the booklet.

Hold up demonstration booklet and point to squares. Then say:

One box has a picture of an apple in it, and the other has a picture of a pear. A big X has been marked on the apple. You will answer the games in this booklet by putting a big X on pictures. Show me how you will mark your answers. Put a big X over the pear.

Check to see that all children mark the pear. Then say:

Very good, Now open your books to the yellow page with a picture of cans on it.

Hold up demonstration booklet opened to the correct page and check to see that all children have that page. Then say:

A Look at the cans on this page. Find the can that is open and put your finger on it.

Check to see that children have the correct answer. Then say:

That's right. The open can has a big X on it. You will do the rest of the games in this book by making a big X on the answer. Try to make an answer to each game. Are there any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the green page. Put an X on the shoe that has laces.

2 Now turn to the pink page. Put an X on the glass that does NOT have milk in it.

3 Now turn to the blue page. If the hands are equal in length, put a big X on both hands. If the hands are not equal in length, put an X on the longer hand.

4 Now turn to the white page. If the books are equal in length, put a big X on both books. If the books are not equal in length, put an X on the longer book.

5 Now turn to the yellow page. If the trees are equal in height, put a big X on both trees. If the trees are not equal in height, put an X on the shorter tree.

6 Now turn to the green page. If the carrots are equal in length, put a big X on both carrots. If the carrots are not equal in length, put an X on the shorter carrot.

7 Now turn to the pink page. If the hammers are equal in length, put a big X on both hammers. If the hammers are not equal in length, put an X on the shorter hammer.

8 Now turn to the blue page. If the boards are equal in length, put a big X on both boards. If the boards are not equal in length, put an X on the longer board.

9 Now turn to the white page. Put an X on the set of things you wear.

10 Now turn to the yellow page. Put an X on the set of things that do NOT hold clothes.

11 Now turn to the green page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

12 Now turn to the pink page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

13 Now turn to the blue page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

14 Now turn to the white page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is smaller in number.

15 Now turn to the yellow page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

16 Now turn to the green page. If the sets are equal in number, put an X on both sets. If the sets are not equal in number, put an X on the set that is larger in number.

17 Now turn to the pink page. Put an X on all of the things that are hot.

18 Now turn to the blue page. Put an X on all of the toys with wheels.

19 Now turn to the white page. Put an X on all of the animals that are NOT farm animals.

20 Now turn to the yellow page. Put an X on all of the things people cook with.

21 Now turn to the green page. Put an X on all of the sets of things people eat with.

22 Now turn to the pink page. Put an X on all of the sets of things that do NOT make loud noises.

23 Now turn to the blue page. Put an X on all of the sets of things used for building.

24 Now turn to the white page. Put an X on all of the sets of things people travel in.

DIRECTIONS FOR TEST III, EQUALIZING OBJECTS ON LENGTH

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Now turn to the yellow page with a picture of boards on it.

Check to see that children have the correct page.

Then say:

A Look at the boards and think about how much you would have to take from the longer board to make the boards equal in length.

Hold up demonstration booklet, point to the marked part of the longer board, and say:

Part of this board has been colored in to show how much would have to be taken away to make the boards equal in length.

Now turn to the blue page with a picture of buildings.

Check to see that children have the correct page.

Then say:

B Think about how much you would add to the shorter building to make the buildings equal in height.

Hold up demonstration booklet, point to marked area above shorter building, and say:

This part has been colored in to show how much must be added to the shorter building to make the buildings equal in height or length. When you answer the games in this book, you will use your crayons to make things equal in length. This is not the kind of book you do your very best coloring in. Try to make an answer to every game.

Are there any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the pink page. Use your crayon to show how much you would add to the shorter licorice stick to make the licorice sticks equal in length.

2 Now turn to the white page. Use your crayon to show how much you would add to the shorter boat to make the boats equal in length.

3 Now turn to the green page. Use your crayon to show how much you would take away from the longer carrot to make the carrots equal in length.

4 Now turn to the yellow page. Use your crayon to show how much you would take away from the longer hammer to make the hammers equal in length.

5 Now turn to the blue page. Use your crayon to show how much you would add to the shorter ladder to make the ladders equal in length.

6 Now turn to the pink page. Use your crayon to show how much you would take away from the longer sled to make the sleds equal in length.

7 Now turn to the white page. Use your crayon to show how much you would add to the shorter chair to make the chairs equal in height.

8 Now turn to the green page. Use your crayon to show how much you would add to the shorter candle to make the candles equal in length.

9 Now turn to the yellow page. Use your crayon to show how much you would take away from the longer toothbrush to make the toothbrushes equal in length.

10 Now turn to the blue page. Use your crayon to show how much you would take away from the longer pencil to make the pencils equal in length.

11 Now turn to the pink page. Use your crayon to show how much you would add to the shorter jeans to make the jeans equal in length.

12 Now turn to the white page. Use your crayon to show how much you would take away from the longer icicle to make the icicles equal in length.

13 Now turn to the green page. Use your crayon to show how much you would add to the shorter hot dog to make the hot dogs equal in length.

14 Now turn to the yellow page. Use your crayon to show how much you would add to the shorter fishing line to make the fishing lines equal in length.

15 Now turn to the blue page. Use your crayon to show how much you would take away from the longer brick to make the bricks equal in length.

16 Now turn to the pink page. Use your crayon to show how much you would add to the shorter table to make the tables equal in length.

17 Now turn to the white page. Use your crayon to show how much you would add to the shorter balloon to make the balloons equal in length.

18 Now turn to the green page. Use your crayon to show how much you would take away from the higher kite to make the kites equally high.

19 Now turn to the yellow page. Use your crayon to show how much you would take away from the taller stairs to make the stairs equal in height.

20 Now turn to the blue page. Use your crayon to show how much you would add to the shorter leash to make the leashes equal in length.

21 Now turn to the pink page. Use your crayon to show how much you would take away from the longer scissors to make the scissors equal in length.

22 Now turn to the white page. Use your crayon to show how much you would take away from the taller chest to make the chests equal in height.

DIRECTIONS FOR TEST IV, EQUALIZING SETS ON NUMEROUSNESS

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Now turn to the yellow page with a picture of glasses and cups on it.

Check to see that children have the correct page. Then say:

A Look at the glasses and cups and think about how many we would have to add to the smaller set to make the number of cups equal to the number of glasses.

Hold up demonstration booklet, point to the circle in the line of glasses, and say:

A circle has been drawn to show the glass we would have to add to make the set of cups and the set of glasses equal in number. Now there is a cup for every glass and a glass for every cup.

Now turn to the blue page with a picture of balls and children.

Check to see that children have the correct page. Then say:

B Think about how many you would have to take away from the larger set to make the number of children equal to the number of balls.

Hold up demonstration booklet, point to X'd ball, and say:

An X has been drawn over this ball to show that we would have to take it away to make the set of children and the set of balls equal in number. Now there is a ball for every child and a child for every ball.

When you answer the games in this book, you will draw circles and X's to make the sets equal in number. Try to make an answer to each game. Do you have any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the pink page. Make a circle for each object you must add to the smaller set to make the set of scissors and the set of crayons equal in number.

2 Now turn to the white page. Make a circle for each object you must add to the smaller set to make the set of trucks and the set of chains equal in number.

3 Now turn to the green page. Make an X over each object you must take away from the larger set to make the set of kites and the set of tops equal in number.

4 Now turn to the yellow page. Make an X over each object you must take away from the larger set to make the set of shoes and the set of socks equal in number.

5 Now turn to the blue page. Make a circle for each object you must add to the smaller set to make the set of hats and the set of jackets equal in number.

6 Now turn to the pink page. Make an X over each object you must take away from the larger set to make the set of books and the set of pencils equal in number.

7 Now turn to the white page. Make a circle for each object you must add to the smaller set to make the set of flowers and the set of bees equal in number.

8 Now turn to the green page. Make a circle for each object you must add to the smaller set to make the set of hammers and the set of nails equal in number.

9 Now turn to the yellow page. Make an X over each object you

must take away from the larger set to make the set of turtles and the set of rabbits equal in number.

10 Now turn to the blue page. Make an X over each object you must take away from the larger set to make the set of shovels and the set of pails equal in number.

11 Now turn to the pink page. Make a circle for each object you must add to the smaller set to make the set of horses and the set of pigs equal in number.

12 Now turn to the white page. Make an X over each object you must take away from the larger set to make the set of lambs and the set of roosters equal in number.

13 Now turn to the green page. Make a circle for each object you must add to the smaller set to make the set of bananas and the set of pears equal in number.

14 Now turn to the yellow page. Make a circle for each object you must add to the smaller set to make the set of spoons and the set of forks equal in number.

15 Now turn to the blue page. Make an X over each object you must take away from the larger set to make the set of bats and the set of dolls equal in number.

16 Now turn to the pink page. Make a circle for each object you must add to the smaller set to make the set of cupcakes and the set of ice cream cones equal in number.

17 Now turn to the white page. Make a circle for each object you must add to the smaller set to make the set of belts and the set of gloves equal in number.

18 Now turn to the green page. Make an X over each object you must take away from the larger set to make the set of teepees and the set of logs equal in number.

19 Now turn to the yellow page. Make an X over each object you must take away from the larger set to make the set of jars and the set of cans equal in number.

20 Now turn to the blue page. Make a circle for each object you must add to the smaller set to make the set of baskets and the set of bowls equal in number.

21 Now turn to the pink page. Make an X over each object you must take away from the larger set to make the set of safety pins and the set of combs equal in number.

22 Now turn to the white page. Make an X over each object you must take away from the larger set to make the set of chairs and the set of tables equal in number.

DIRECTIONS FOR TEST V, ORDERING MORE THAN TWO OBJECTS OR SETS

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Now turn to the green page with a picture of people on it to see how to do the games in this book.

Check to see that children have the correct page. Then say:

The people on this page are different heights. How would they stand in order by height? On the inside of the page there is a place for each person.

Hold up booklet and show places for figures. Then say:

A The tallest person, the astronaut, is standing in order by height. Put your finger on the person who is next tallest, the one who will stand next to the astronaut.

Check to see that children indicate the correct figure. Then say:

That's right. The next person, the girl wearing a coat and boots, has been marked with a big X. You will make a big X to answer the games in this booklet. Try to make an answer to every game.

Do you have any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the pink page. Think about putting the rakes in order by length. Put an X on the shortest rake.

2 Now turn to the blue page. Think about putting the tables in order by height. Put an X on the tallest table.

3 Now turn to the white page. Think about putting the candles in order by height. Put an X on the tallest candle.

4 Now turn to the yellow page. Think about putting the bones in order by length. Put an X on the shortest bone.

5 Now turn to the green page. At the top of the page there is a place for every spoon, a place to put the spoons in order by length. One has been put in order. Put an X on the spoon at the bottom of the page that goes next in order by length.

6 Now turn to the pink page. On the inside of the page there is a place for each flagpole, a place to put the flagpoles in order by height. One has been put in order. Put an X on the flagpole that goes next in order by height.

7 Now turn to the blue page. At the top of the page there is a place for each pencil, a place to put the pencils in order by length. Some have been put in order. Put an X on the pencil at the bottom of the page that goes next in order by length.

8 Now turn to the white page. At the top of the page there is a place for the cars and trucks to be put in order by length. Some have been put in order. Put an X on the car at the bottom of the page that goes next in order by length.

9 Now turn to the yellow page. At the top of the page there is a place to put the candysticks in order by length. Some of them have been put in order. Put an X on the candystick at the bottom of the page that goes next in order by length.

10 Now turn to the green page. On the inside of the page there is a place to put each trashcan in order by height. One trashcan has been put in order. Put an X on the trashcan that goes next in order by height.

11 Now turn to the pink page. On the inside of the page there is

a place to put each building in order by height. Some of the buildings have been put in order. Put an X on the building that goes next in order by height.

12 Now turn to the blue page. On the inside of the page there is a place to put each book in order by height. One book has been put in order. Put an X on the book that goes next in order by height.

13 Now turn to the white page. Think about putting the sets in order by number. Put an X on the set that is smallest in number.

14 Now turn to the yellow page. Think about putting the sets in order by number. Put an X on the set that is largest in number.

15 Now turn to the green page. Think about putting the sets in order by number. Put an X on the set that is largest in number.

16 Now turn to the pink page. Think about putting the sets in order by number. Put an X on the set that is smallest in number.

17 Now turn to the blue page. At the top of the page there is a place to put the sets in order by number. One set has been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

18 Now turn to the white page. At the top of the page there is a place to put the sets in order by number. One set has been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

19 Now turn to the yellow page. At the top of the page there is a place to put the sets in order by number. One set has been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

20 Now turn to the green page. At the top of the page there is a place to put the sets in order by number. Some of the sets have been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

21 Now turn to the pink page. At the top of the page there is a place to put the sets in order by number. Some of the sets have been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

22 Now turn to the blue page. At the top of the page there is a to put the sets in order by number. One of the sets has been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

23 Now turn to the white page. At the top of the page there is a place to put the sets in order by number. Some of the sets have been put in order. Put an X on the set at the bottom of the page that goes next in order by number.

DIRECTIONS FOR TEXT VI, COUNTING AND RECOGNIZING NUMERALS I

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the space provided. Then say:

Open your booklets to the blue page with a picture of a chain on it.

Check to see that children have the correct page.

Then say:

A Look at the chain on this page. How long would the chain be if it were three links long?

A circle has been drawn around a chain three links long. You will answer games in this booklet by drawing circles like this one. Try to make an answer to each game. Do you have any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the white page with a picture of train cars on it. How long would the line of train cars be if it were two cars long? Draw a circle around a line that is two train cars long.

2 Now turn to the yellow page. How tall would the stack of bowls be if it were four bowls high? Draw a circle around a stack that is four bowls high.

3 Now turn to the green page. Draw a circle around a row of buildings that is one building long.

4 Now turn to the pink page. Draw a circle around a brick post that is six bricks high.

5 Now turn to the blue page. Draw a circle around a set of eight pieces of pie.

6 Now turn to the white page. Draw a circle around a set of seven gloves.

7 Now turn to the yellow page. Draw a circle around a set of five crayons.

8 Now turn to the green page. Draw a circle around a set of one drum.

9 Now turn to the pink page. Look at the numeral on this page. Draw a circle around a loaf of bread that is as many slices long as the numeral says.

10 Now turn to the blue page. Look at the numeral on this page. Draw a circle around a fence that is as many boards long as the numeral says.

11 Now turn to the white page. Look at the numeral on this page. Draw a circle around a ladder that is as many steps long as the numeral says.

12 Now turn to the yellow page. Look at the numeral on this page. Draw a circle to show a pea pod that is as many peas long as the numeral says.

13 Now turn to the green page. Look at the numeral on this page. Draw a circle to make a set that has as many wheels as the numeral says.

14 Now turn to the pink page. Look at the numeral on this page. Draw a circle to make a set that has as many flags as the numeral says.

15 Now turn to the blue page. Look at the numeral on this page. Draw a circle to make a set that has as many bags as the numeral says.

16 Now turn to the white page. Look at the numeral on this page. Draw a circle to make a set that has as many fish as the numeral says.

B Now turn to the yellow page. There are apples drawn on this page and a row of numerals along the bottom.

Hold up demonstration booklet and point to the apples and the row of numerals. Then say:

Put your finger on the numeral that tells how many apples are drawn on this page.

Check to see that children indicate 8. Then say:

Very good. A circle has been drawn around the numeral that tells how many apples there are on this page. You will answer the rest of the games in this booklet by drawing circles around numerals. Try to make an answer to every game.

17 Now turn to the green page. Draw a circle around the numeral that tells how many shelves high the bookcase is.

18 Now turn to the pink page. Draw a circle around the numeral that tells how many windows long the building is.

19 Now turn to the blue page. Draw a circle around the numeral that tells how many doors long the cupboard is.

20 Now turn to the white page. Draw a circle around the numeral that tells how many cushions long the sofa is.

21 Now turn to the yellow page. Draw a circle around the numeral that tells how many pencils long the book is.

22 Now turn to the green page. Draw a circle around the numeral that tells how many boats long the pier is.

23 Now turn to the pink page. Draw a circle around the numeral that tells how many dogs there are on this page.

24 Now turn to the blue page. Draw a circle around the numeral that tells how many seahorses there are on this page.

25 Now turn to the white page. Draw a circle around the numeral that tells how many toothbrushes there are on this page.

26 Now turn to the yellow page. Draw a circle around the numeral that tells how many cars there are on this page.

27 Now turn to the green page. Draw a circle around the numeral that tells how many pears there are on this page.

28 Now turn to the pink page. Draw a circle around the numeral that tells how many globes there are on this page.

DIRECTIONS FOR TEST VII, COUNTING AND RECOGNIZING NUMERALS II

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the spaces provided. Then say:

Open your booklets to the blue page with a picture of a chain on it.

Check to see that children have the correct page.

Then say:

A Look at the chain on this page. How long would the chain be if it were three links long?

A circle has been drawn around a chain three links long.

You will answer games in this booklet by drawing circles

14 Now turn to the pink page. Look at the numeral on this page. Draw a circle to make a set that has as many flags as the numeral says.

15 Now turn to the blue page. Look at the numeral on this page. Draw a circle to make a set that has as many bags as the numeral says.

16 Now turn to the white page. Look at the numeral on this page. Draw a circle to make a set that has as many fish as the numeral says.

B Now turn to the yellow page. There are apples drawn on this page and a row of numerals along the bottom.

Hold up demonstration booklet and point to the apples and the row of numerals. Then say:

Put your finger on the numeral that tells how many apples are drawn on this page.

Check to see that children indicate 8. Then say:

Very good. A circle has been drawn around the numeral that tells how many apples there are on this page. You will answer the rest of the games in this booklet by drawing circles around numerals. Try to make an answer to every game.

17 Now turn to the green page. Draw a circle around the numeral that tells how many shelves high the bookcase is.

18 Now turn to the pink page. Draw a circle around the numeral that tells how many windows long the building is.

19 Now turn to the blue page. Draw a circle around the numeral that tells how many doors long the cupboard is.

20 Now turn to the white page. Draw a circle around the numeral that tells how many cushions long the sofa is.

21 Now turn to the yellow page. Draw a circle around the numeral that tells how many pencils long the book is.

22 Now turn to the green page. Draw a circle around the numeral that tells how many boats long the pier is.

23 Now turn to the pink page. Draw a circle around the numeral that tells how many dogs there are on this page.

24 Now turn to the blue page. Draw a circle around the numeral that tells how many seahorses there are on this page.

25 Now turn to the white page. Draw a circle around the numeral that tells how many toothbrushes there are on this page.

26 Now turn to the yellow page. Draw a circle around the numeral that tells how many cars there are on this page.

27 Now turn to the green page. Draw a circle around the numeral that tells how many pears there are on this page.

28 Now turn to the pink page. Draw a circle around the numeral that tells how many globes there are on this page.

DIRECTIONS FOR TEST VII, COUNTING AND RECOGNIZING NUMERALS II

When all children are seated, say:

Today we are going to do some games. There are questions to answer in the games. Try to make an answer to each game. If you're not sure about the answer to a game, you should guess. Now I'm going to give each of you a booklet. Please don't look inside until I tell you.

Distribute booklets and be sure each child has a crayon. If you have not previously written identifying information on the booklets, have the children write their names in the spaces provided. Then say:

Open your booklets to the blue page with a picture of a chain on it.

Check to see that children have the correct page.

Then say:

A Look at the chain on this page. How long would the chain be if it were three links long?

A circle has been drawn around a chain three links long.

You will answer games in this booklet by drawing circles like this one. Try to make an answer to each game. Do you have any questions?

Answer questions by rereading a sentence or two from the directions. Then say:

1 Now turn to the white page with a picture of a line of cars. How long would the line of cars be if it were eight cars long? Draw a circle around a line that is eight cars long.

2 Now turn to the yellow page. How tall would the stack of books be if it were two books high? Draw a circle around a stack that is two books high.

3 Now turn to the green page. Draw a circle around a row of chairs that is zero chairs long.

4 Now turn to the pink page. Draw a circle to show a chest that is nine drawers tall.

5 Now turn to the blue page. Draw a circle around a set of six hats.

6 Now turn to the white page. Draw a circle around a set of five cupcakes.

7 Now turn to the yellow page. Draw a circle around a set of three forks.

8 Now turn to the green page. Draw a circle around a set of ten bells.

9 Now turn to the pink page. Look at the numeral on this page. Draw a circle around a stack of blocks that is as many blocks tall as the numeral says.

10 Now turn to the blue page. Look at the numeral on this page. Draw a circle to show a bus as many windows long as the numeral says.

11 Now turn to the white page. Look at the numeral on this page. Draw a circle around a bench as many flower pots long as the numeral says.

12 Now turn to the yellow page. Look at the numeral on this page. Draw a circle around a folding door as many pieces long as the numeral says.

13 Now turn to the green page. Look at the numeral on this page. Draw a circle to make a set that has as many seals as the numeral says.

14 Now turn to the pink page. Look at the numeral on this page. Draw a circle to make a set that has as many pitchers as the numeral says.

15 Now turn to the blue page. Look at the numeral on this page. Draw a circle to make a set that has as many paper bags as this numeral says.

16 Now turn to the white page. Look at the numeral on this page. Draw a circle to make a set that has as many baseball bats as this numeral says.

B Now turn to the yellow page. There are apples drawn on this page and a row of numerals along the bottom.

> Hold up demonstration booklet and point to the apples and the row of numerals. Then say:

Put your finger on the numeral that tells how many apples are drawn on this page.

> Check to see that children indicate 8. Then say:

Very good. A circle has been drawn around the numeral that tells how many apples there are on this page. You will answer the rest of the games in this booklet by drawing circles around numerals. Try to make an answer to every game.

17 Now turn to the green page. Draw a circle around the numeral that tells how many books long the bookshelf is.

18 Now turn to the pink page. Draw a circle around the numeral that tells how many forks long the table is.

19 Now turn to the blue page. Draw a circle around the numeral that tells how many boxes long the wagon is.

20 Now turn to the white page. Draw a circle around the numeral that tells how many train cars there are on the track.

21 Now turn to the yellow page. Draw a circle around the numeral that tells how many scoops tall the ice cream is.

22 Now turn to the green page. Draw a circle around the numeral that tells how many sections long the bridge is.

23 Now turn to the pink page. Draw a circle around the numeral that tells how many baskets there are on this page.

24 Now turn to the blue page. Draw a circle around the numeral that tells how many shovels there are on this page.

25 Now turn to the white page. Draw a circle around the numeral that tells how many sleds there are on this page.

26 Now turn to the yellow page. Draw a circle around the numeral that tells how many coathooks there are on this page.

27 Now turn to the green page. Draw a circle around the numeral that tells how many birdhouses there are on this page.

28 Now turn to the pink page. Draw a circle around the numeral that tells how many teeter-totters there are on this page.

APPENDIX B

Tests Administered in Item Tryouts

Complete copies of the tests have been omitted from this report.
They are available in the thesis or from the R & D Center

Samples of each item type may be found in Appendix C.

APPENDIX C

Materials Distributed for Content Validity Study

WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
1404 REGENT STREET
MADISON, WISCONSIN 53706
PHONE 262-4901 AREA 608

Dear__________:

Thank you for agreeing to help us in the development of a test of the material covered in Developing Mathematical Processes Arithmetic Book I. Your knowledge and experience with Processes in your class this year can help to make the test a more useful instrument. Information that you may need about the test--its intended use, its planned appearance--is given below.

The Arithmetic Survey is intended to provide an overall picture of each child's understanding of the mathematical content of Arithmetic Book I, with the exception of making representations. Results could be used by a first-grade teacher in the fall to group her class for mathematics instruction. The content of review that might be necessary when Book I is completed during the school year could also be determined.

Group administration of the test in half-hour segments is planned with a separate test booklet provided for each child for each segment. There will be one test item on each page and pages will vary in color so the test administrator can tell at a glance whether all children have the proper page. All directions will be read aloud to the children. Results for each child are to be plotted on a profile. By studying the profiles, teachers using Developing Mathematical Processes will be able to determine which children are ready to learn new concepts and skills and which children have weaknesses that should be more closely investigated.

In a testing situation the children will see sample items and work through such items before they respond to the test items themselves. The enclosed review package contains only representative test items. Directions read by the test administrator appear at the top of each page with the item those directions refer to immediately below; at the bottom of the first page of each group of items is a question we would like you to answer. Please make any other comments you wish, on the materials themselves or on separate pages.

Please try to complete your review by Monday, May 18; a business reply envelope is enclosed for the return of your comments.

Again, thank you for agreeing to help in test development.

Sincerely,

Deborah M. Stewart
Project Assistant

WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

THE UNIVERSITY OF WISCONSIN
1404 REGENT STREET
MADISON, WISCONSIN 53706
PHONE 262-4901 AREA 608

To: AMI Project Staff

From: Debbie Stewart

Re: Group test for Arithmetic Book I

Please look over the attached sample test. Use one of these response sheets to answer the question below the first item in each group of sample items and to make any other comments.

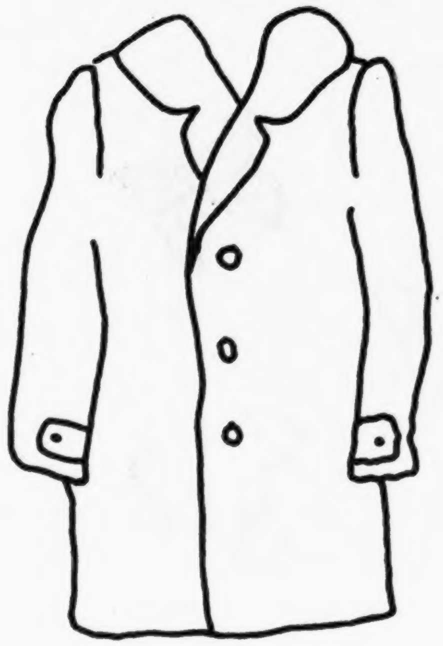
Item Group	Answer		Comments
	Yes	No	
I			
II			
III			
IV			
V			
VI			
VII			
VIII			
IX			
X			
XI			

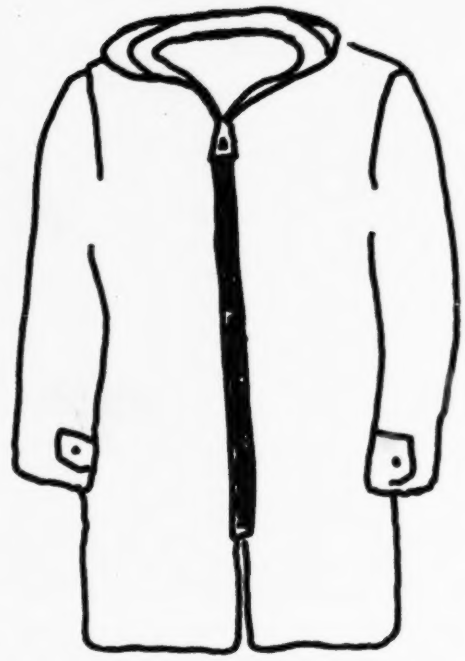
The construction and use of representations was not tested--except that such a test consists of pictorial representations.

Are there other objectives omitted that you feel should have been included? Which ones?

TEACHER READS:

Put an X on the coat that has buttons.

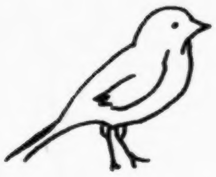


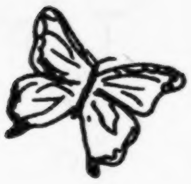


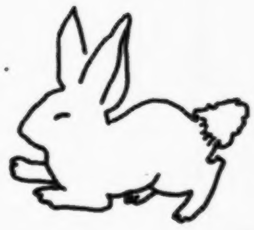
Would you accept correct responses to items like these as evidence that a child can differentiate among properties of objects and separate objects into two sets, positive and negative instances of a single property value?

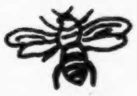
TEACHER READS:

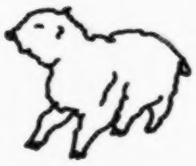
Put an X on all of the things that do NOT fly.





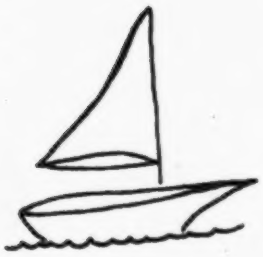


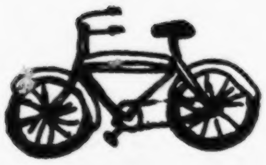


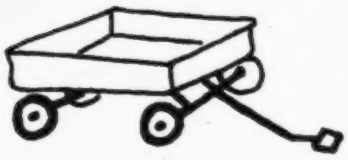


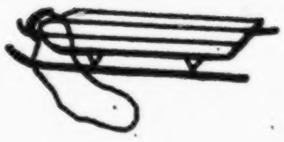
TEACHER READS:

Put an X on all of the toys with wheels.



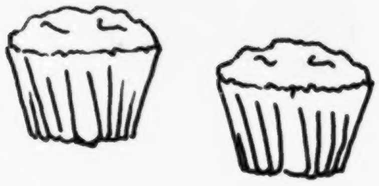


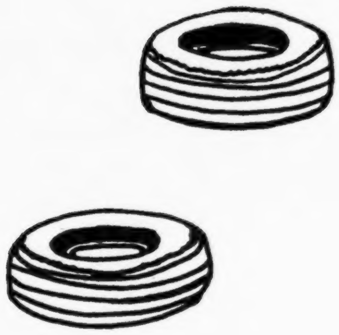




TEACHER READS:

Put an X on the set of things you eat.

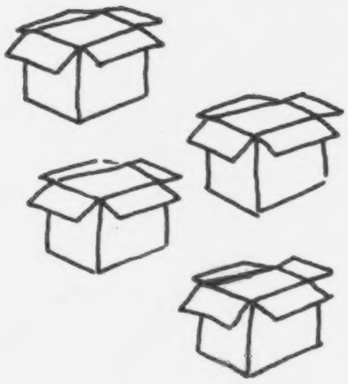


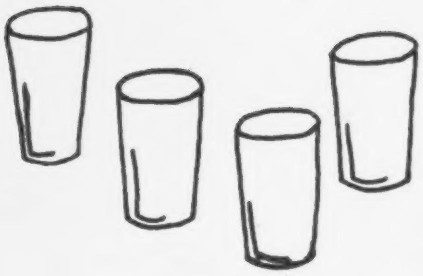


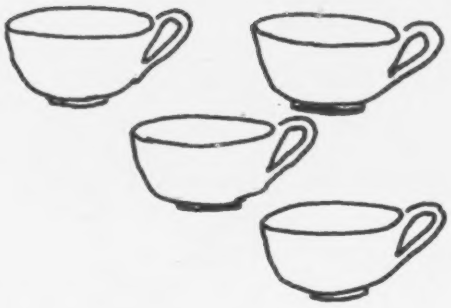
Would you accept correct responses to items like these as evidence that a child can separate sets into two groups, positive and negative instances of a single property value?

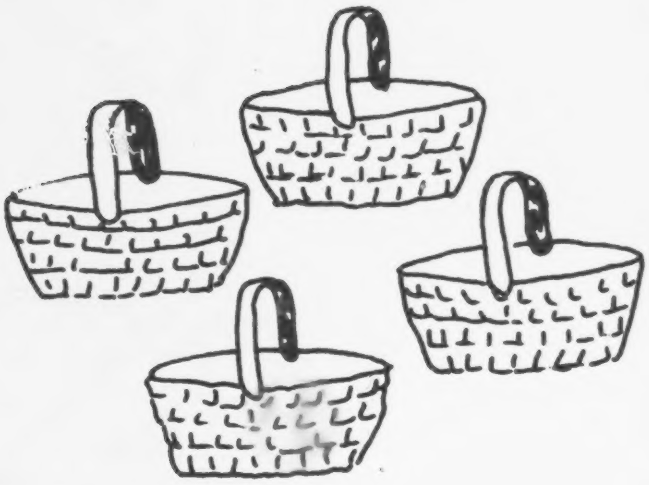
TEACHER READS:

Put an X on all of the sets of things you do NOT drink from.



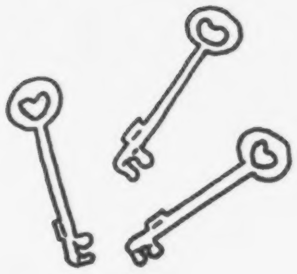


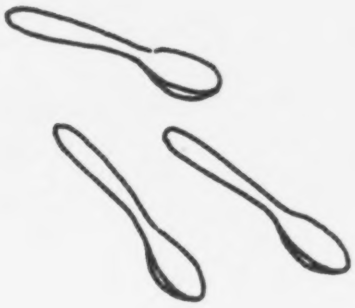


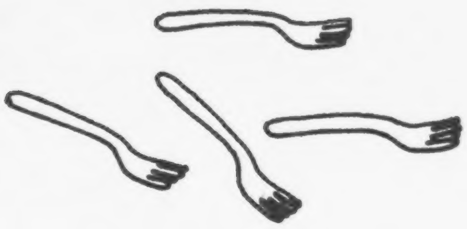


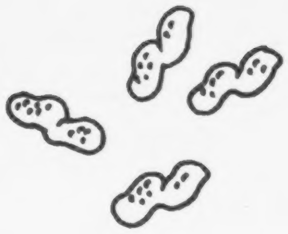
TEACHER READS:

Put an X on all of the sets of things people eat with.



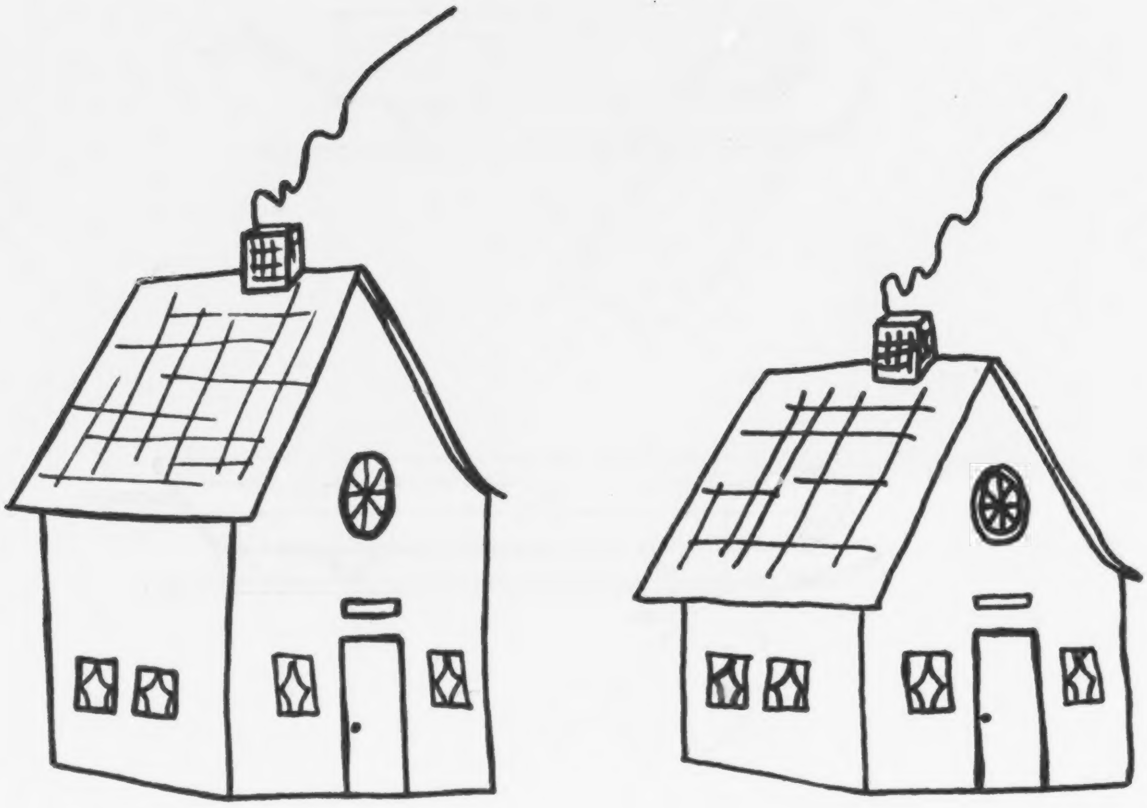






TEACHER READS:

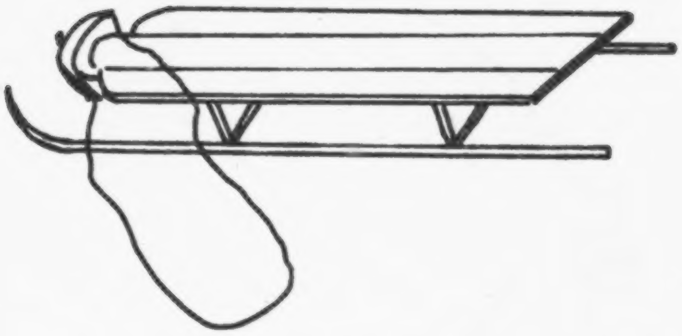
Put an X on the shorter house. If the houses are equal in height, put an X on both houses.

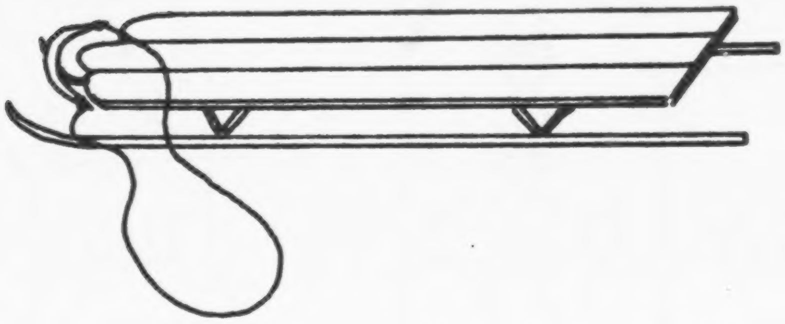


Would you accept correct responses to items like these as evidence that a child can determine whether two objects are the same length and "state" the length relationship between two objects of unequal length?

TEACHER READS:

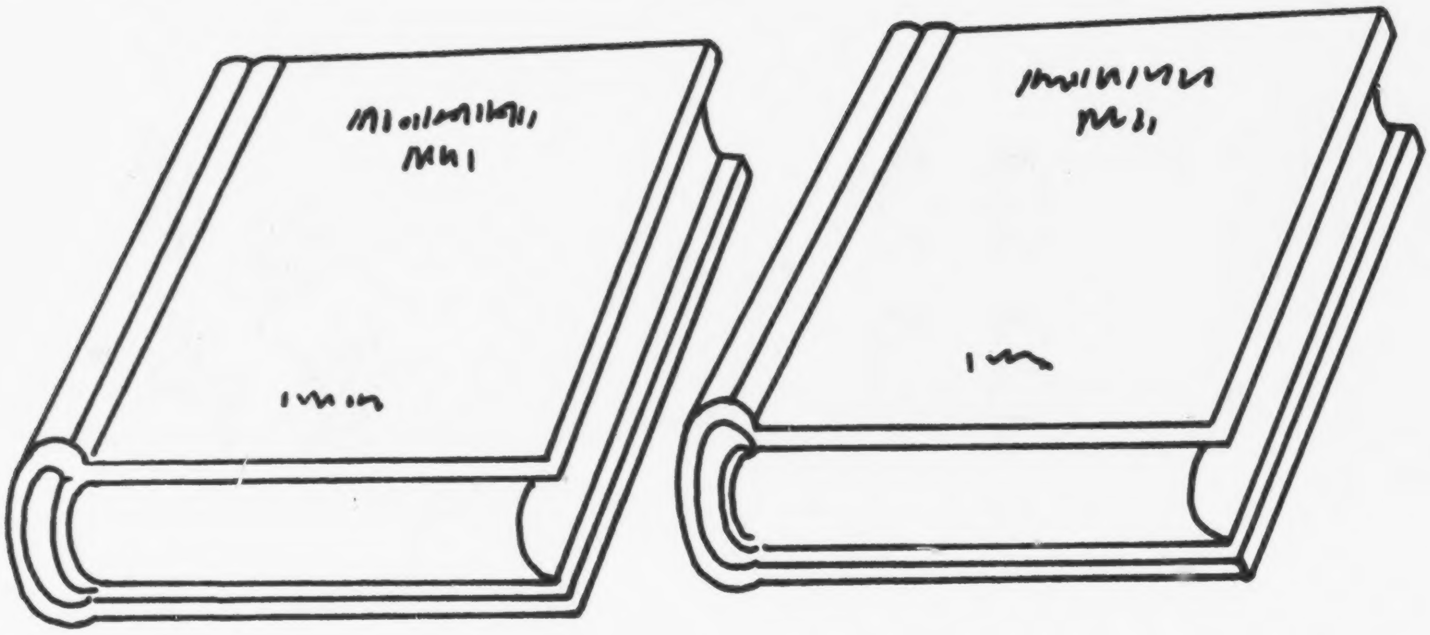
Put an X on the longer sled. If the sleds are equal in length, put an X on both sleds.





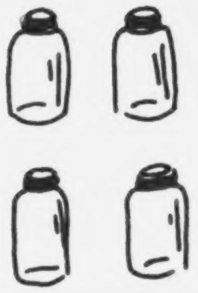
TEACHER READS:

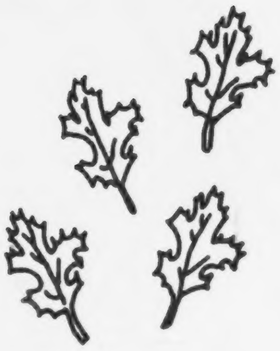
Put an X on the longer book. If the books are equal in length, put a big X on both books.



TEACHER READS:

Put an X on the set that is larger in number. If the sets are equal in number, put an X on both sets.

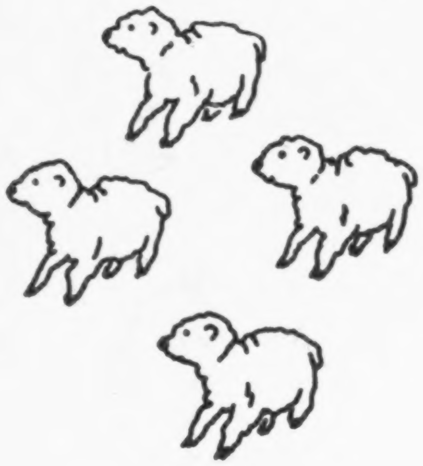


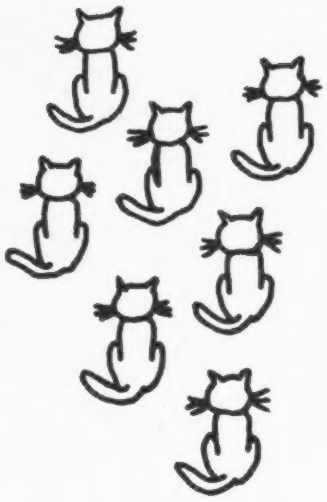


Would you accept correct responses to items like these as evidence that a child can determine whether the numerousness of two sets is the same and "state" the numerousness relationship which exists between two sets of unequal size?

TEACHER READS:

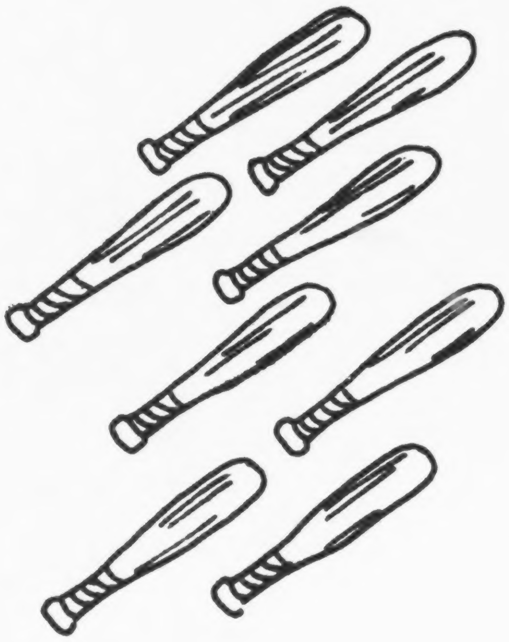
Put an X on the set that is smaller in number. If the sets are equal in number, put an X on both sets.

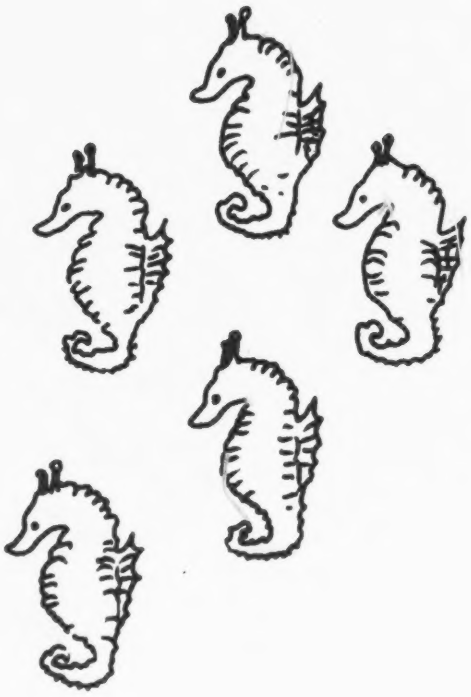




TEACHER READS:

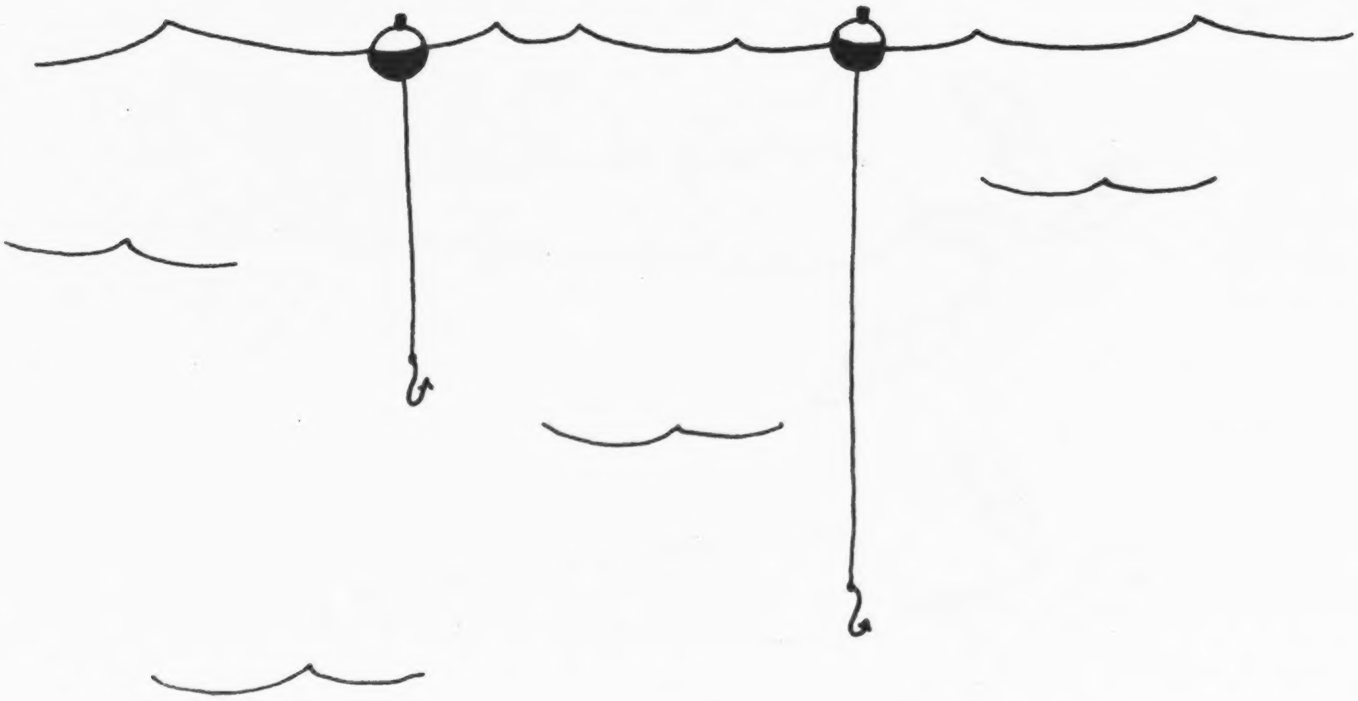
Put an X on the set that is larger in number. If the sets are equal in number, put an X on both sets.





TEACHER READS:

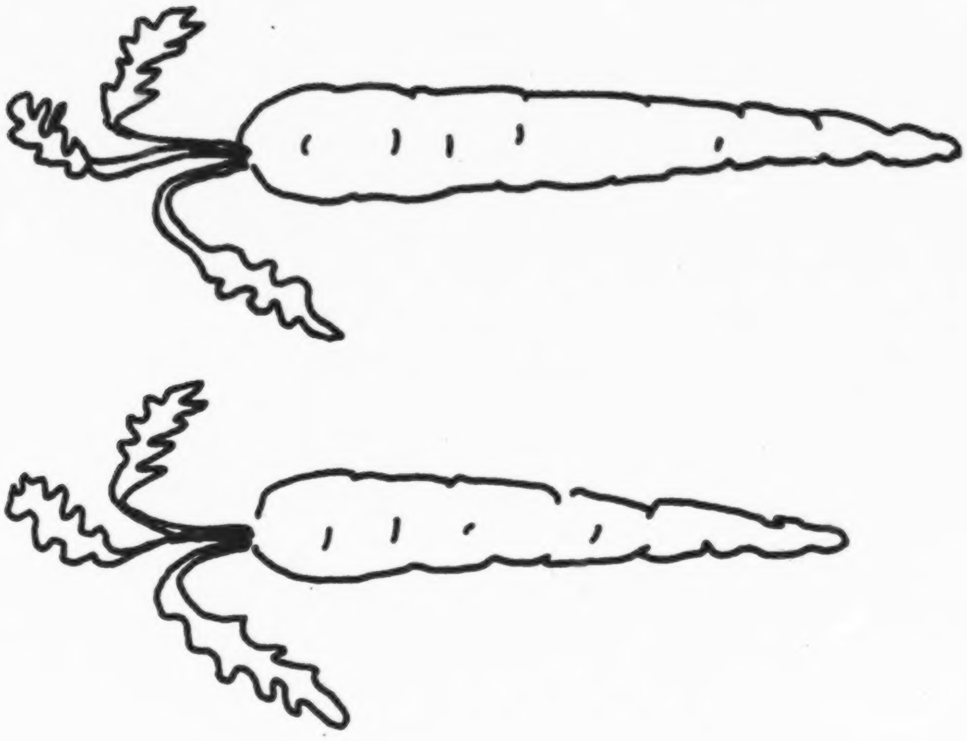
Color to show how much you would add to the shorter fishing line to make the fishing lines equal in length.



Would you accept correct responses to items like these as evidence that a child can equalize the length of objects which are unequal, either by "taking away" from the longer object or by "adding to" the shorter object?

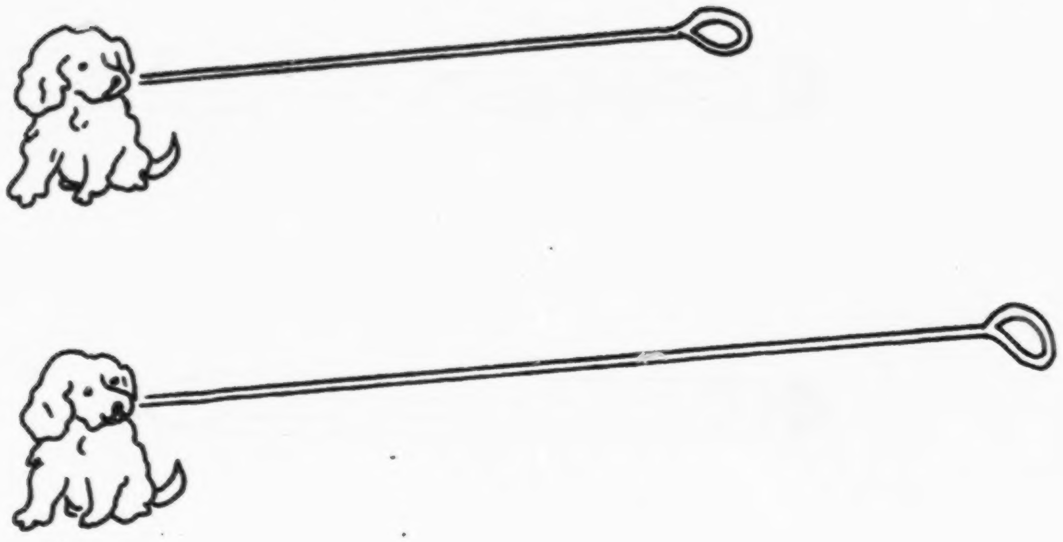
TEACHER READS:

Color to show how much you would take away from the longer carrot to make the carrots equal in length.



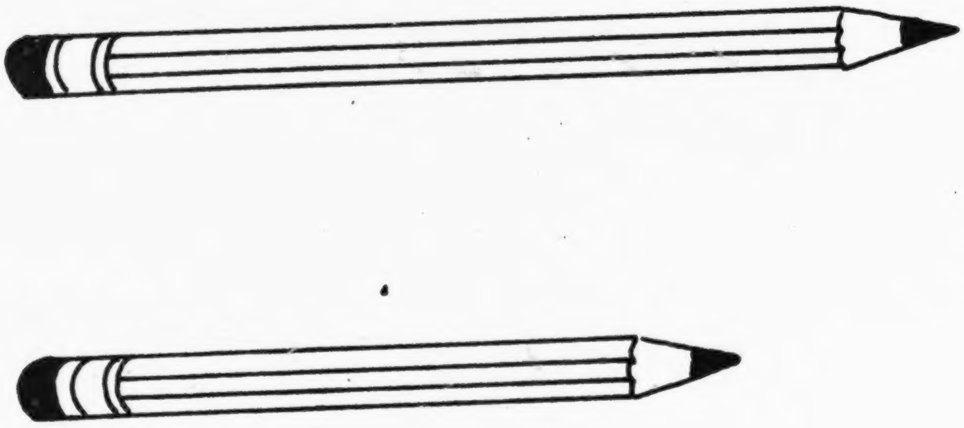
TEACHER READS:

Color to show how much you would add to the shorter leash to make the leashes equal in length.



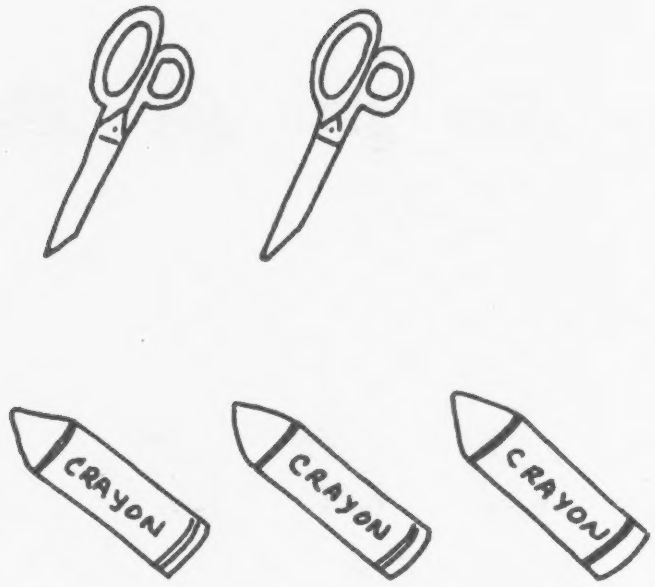
TEACHER READS:

Color to show how much you would take away from the longer pencil to make the pencils equal in length.



TEACHER READS:

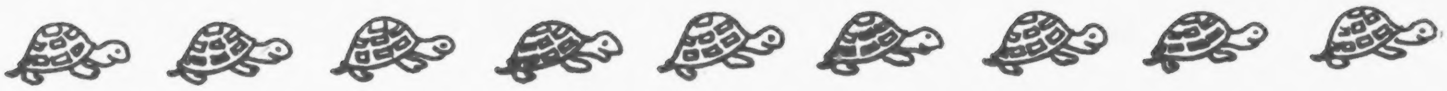
Make a circle for each object you must add to the smaller set to make the set of scissors and the set of crayons equal in number.

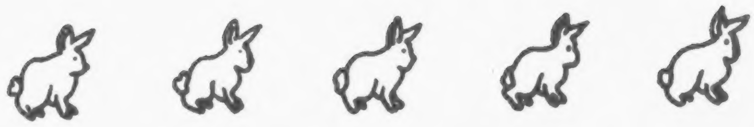


Would you accept correct responses to items like these as evidence that a child can equalize any two sets of objects which are unequal in number by either adding more objects to the smaller set or taking objects away from the larger set?

TEACHER READS:

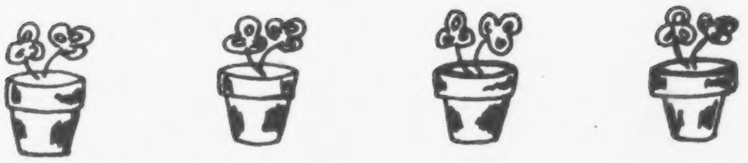
Make an X over each object you must take away from the larger set to make the set of turtles and the set of rabbits equal in number.

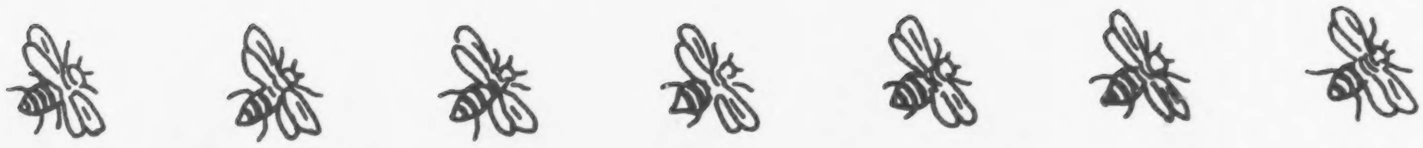




TEACHER READS:

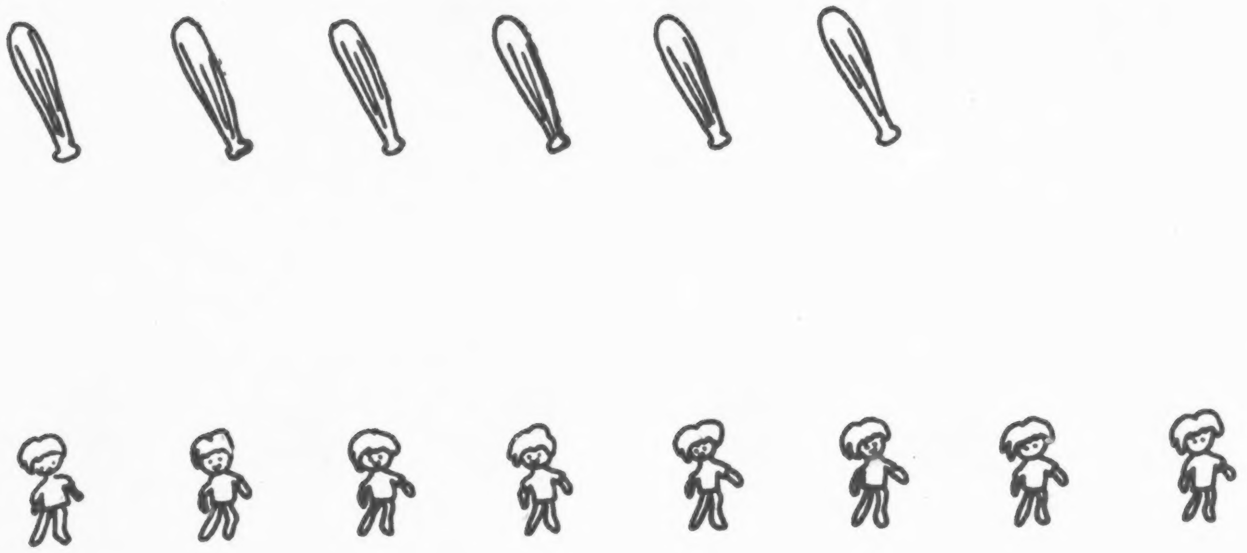
Make a circle for each object you must add to the smaller set to make the set of flowers and the set of bees equal in number.





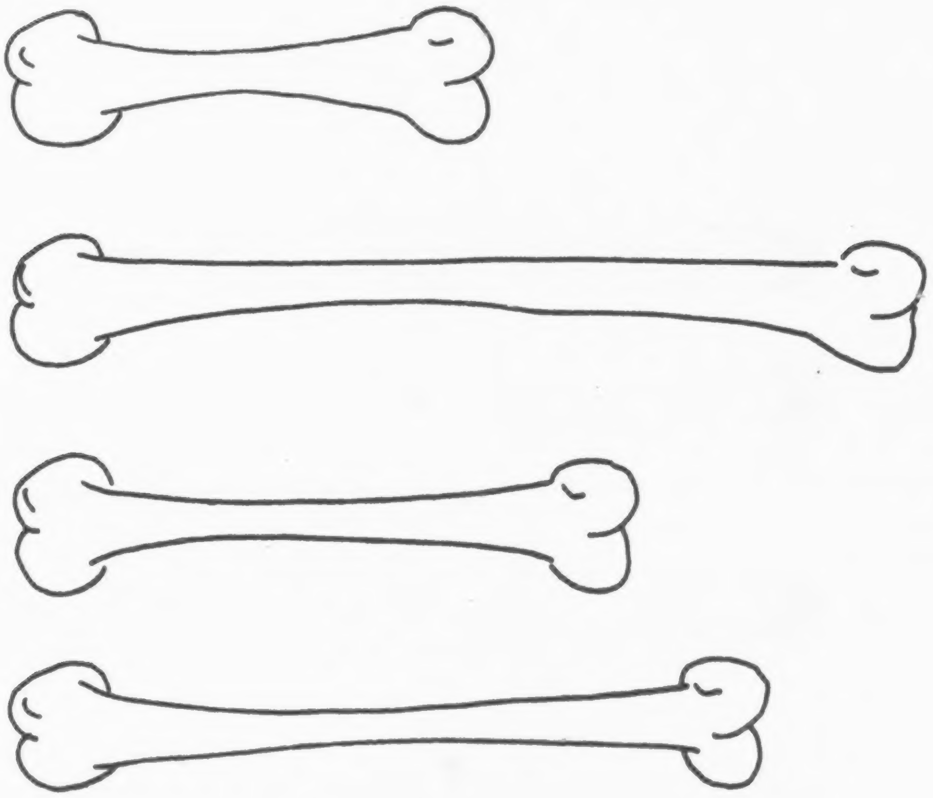
TEACHER READS:

Make an X over each object you must take away from the larger set to make the set of bats and the set of dolls equal in number.



TEACHER READS:

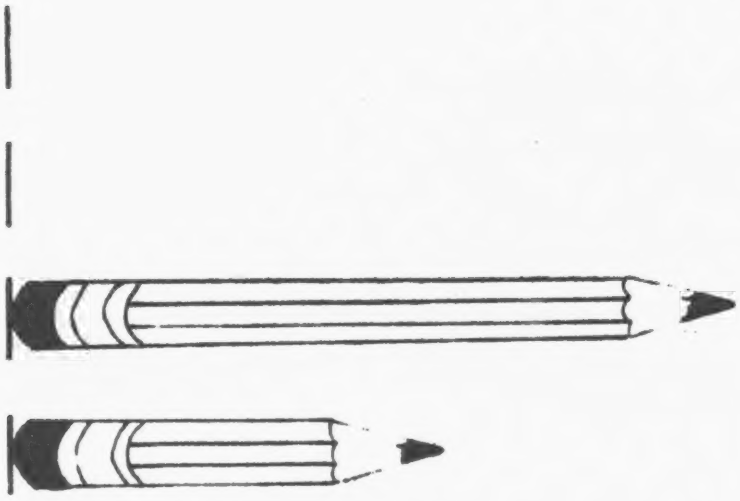
Think about putting the bones in order by length. Put an X on the shortest bone.

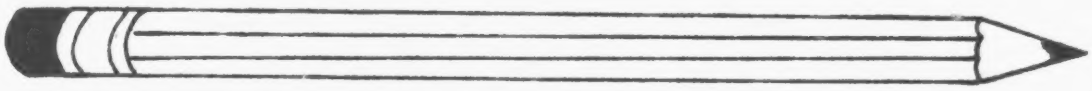


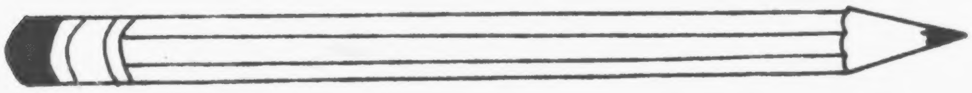
Would you accept correct responses to items like these as evidence that a child can order more than two objects from shortest to longest or longest to shortest?

TEACHER READS:

At the top of the page there is a place for each pencil, a place to put the pencils in order by length. Some pencils have been put in order. Put an X on the pencil that goes next in order by length.

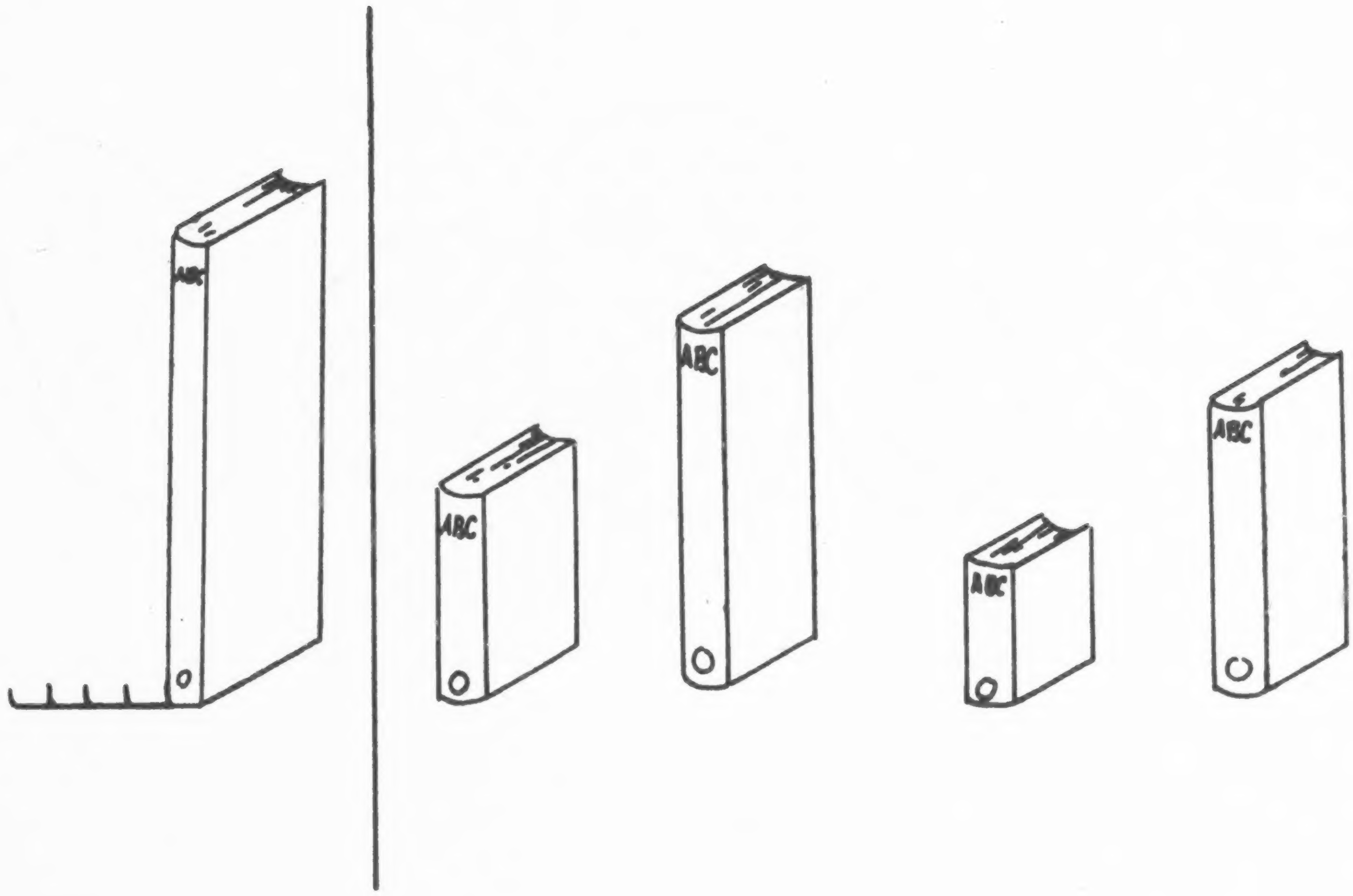






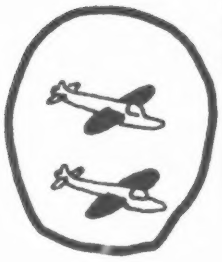
TEACHER READS:

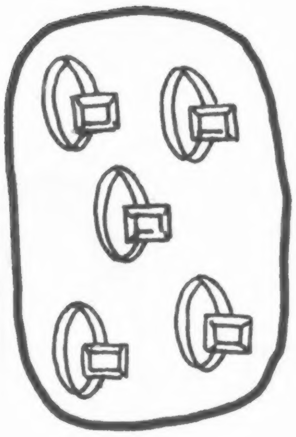
On the inside of the page there is a place to put each book in order by height. One book has been put in order. Put an X on the book that goes next in order by height.

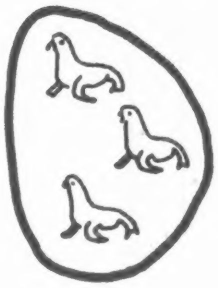


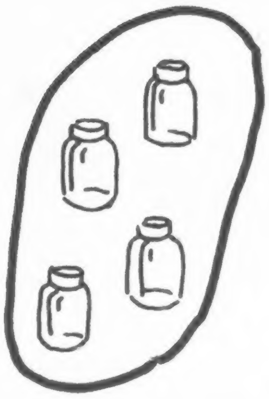
TEACHER READS:

Think about putting the sets in order by number. Put an X on the set that is largest in number.





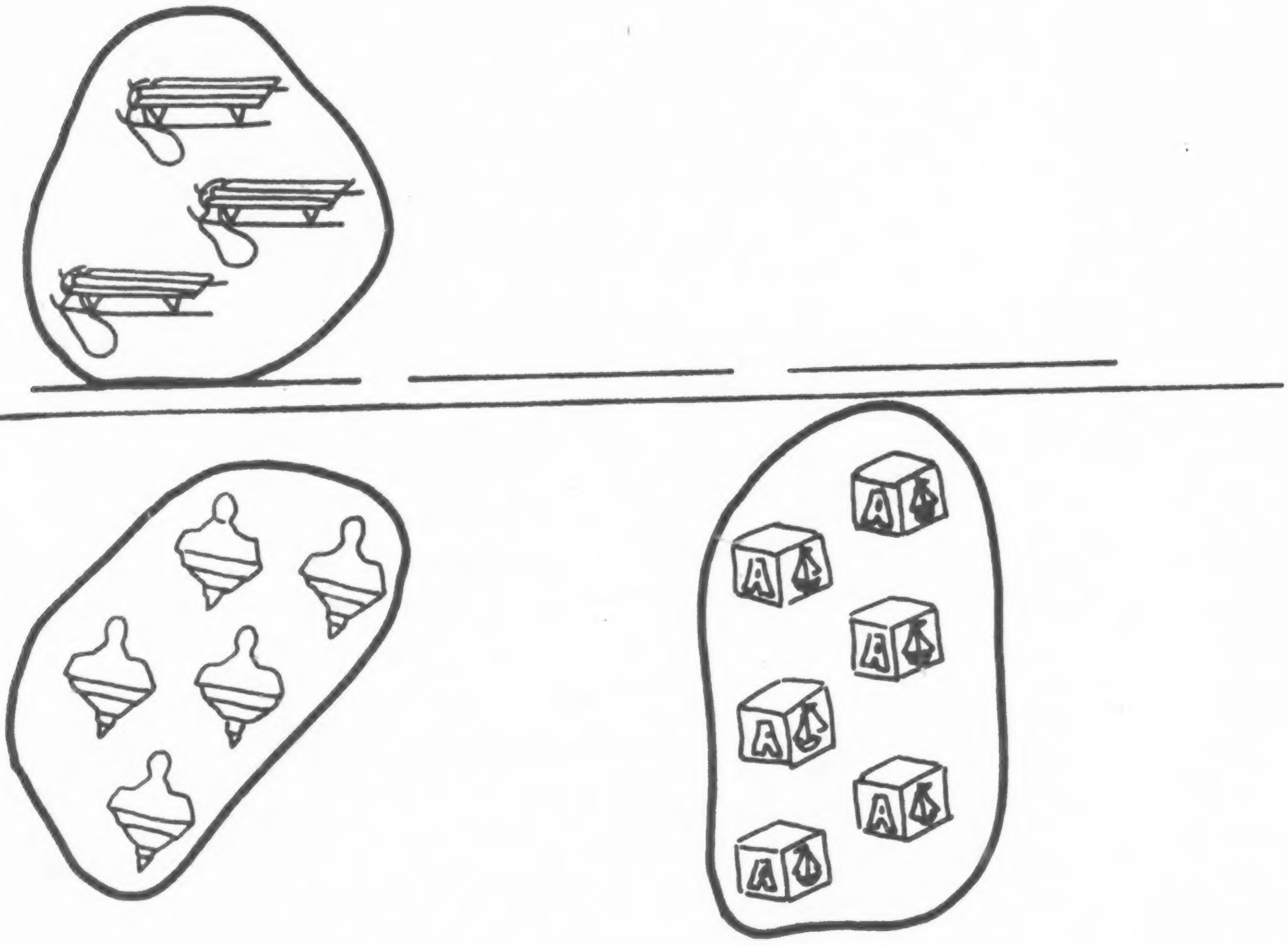




Would you accept correct responses to items like these as evidence that a child can order more than two sets on numerousness from largest to smallest or smallest to largest?

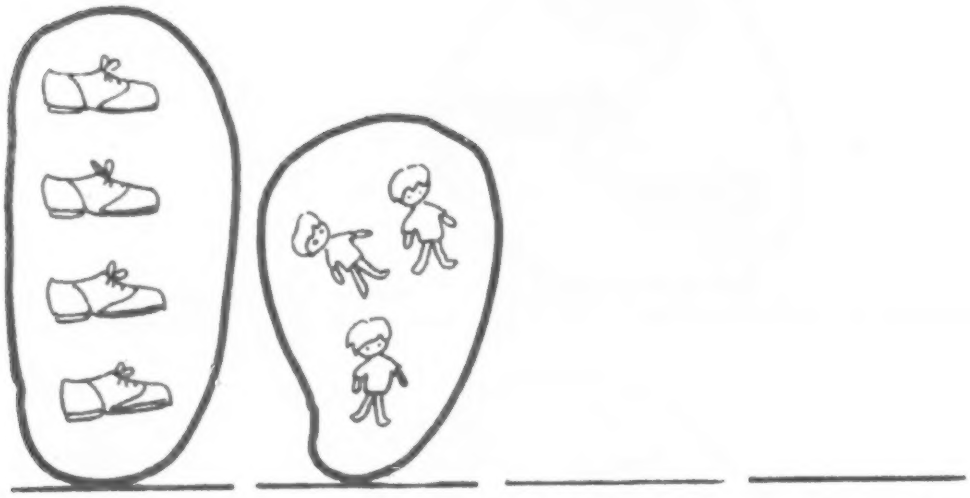
TEACHER READS:

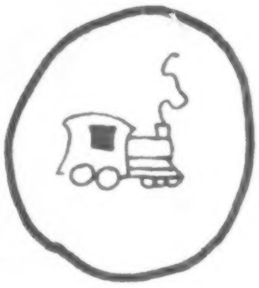
At the top of the page there is a place to put the sets in order by number. One set has been put in order. Put an X on the set that goes next in order by number.

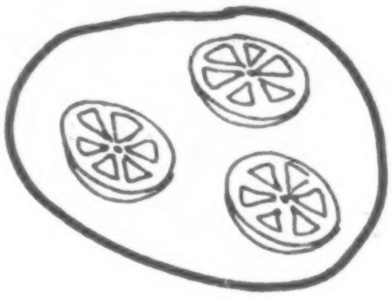


TEACHER READS:

At the top of the page there is a place to put the sets in order by number. Some of the sets have been put in order. Put an X on the set that goes next in order by number.

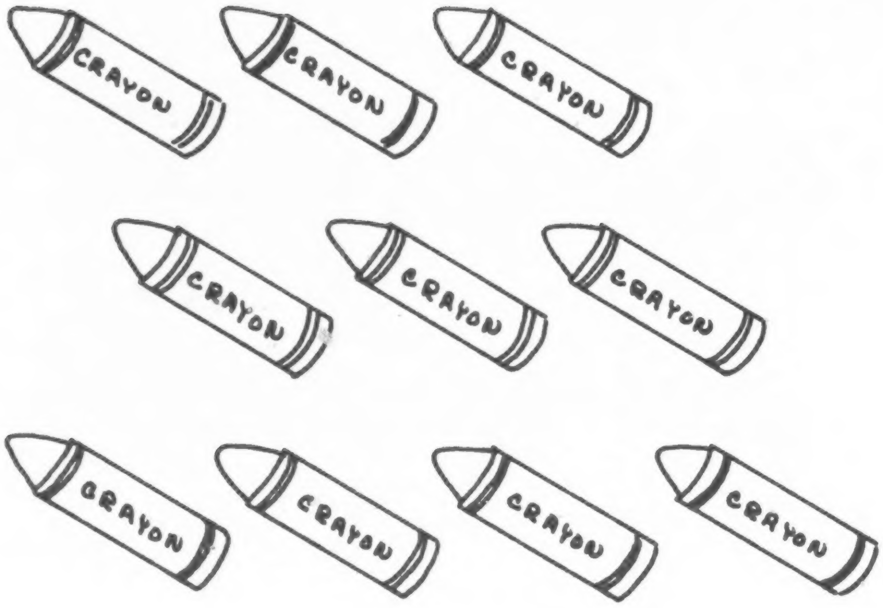






TEACHER READS:

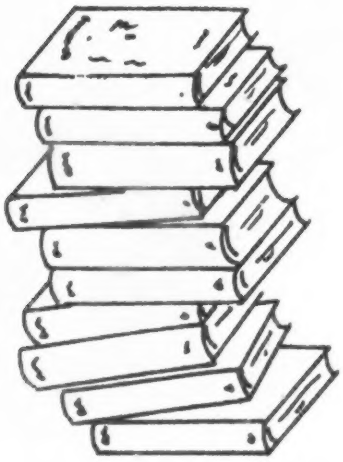
Draw a circle to show a set of five crayons.



Would you accept correct responses to items like these as evidence that a child can illustrate or display a set of objects the number of which is specified by a stated number from zero to ten?

TEACHER READS:

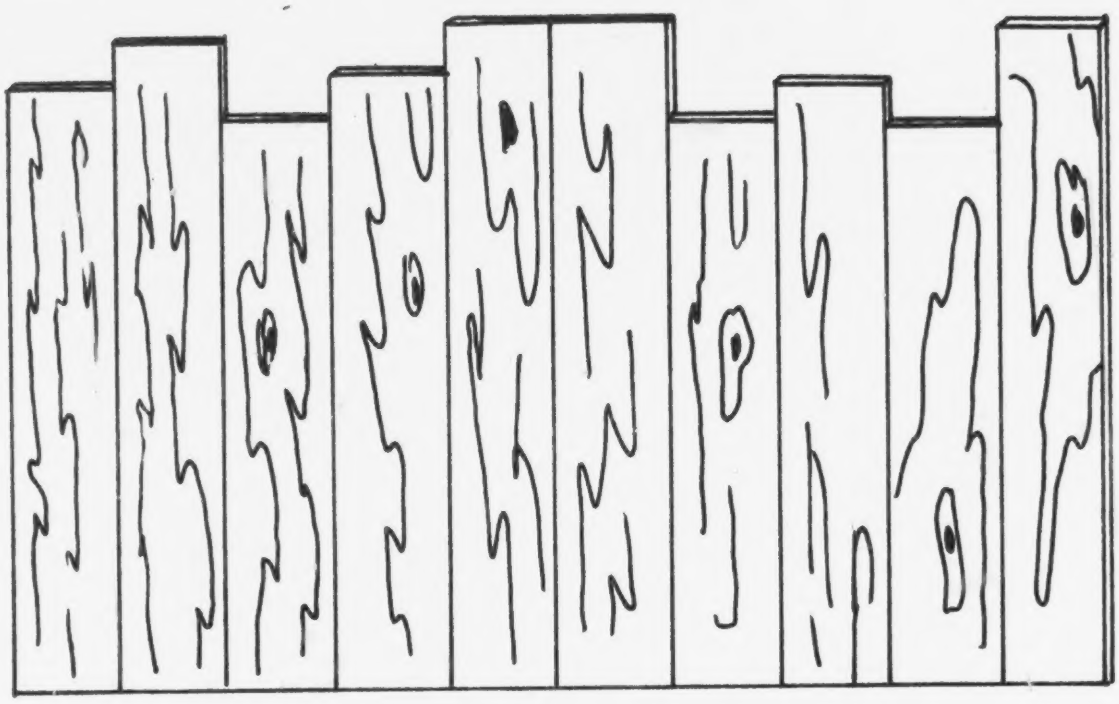
How tall would the stack of books be if it were two books high? Draw a circle to show a stack two books high.



TEACHER READS:

Look at the numeral on this page. Draw a circle to show a fence as many boards long as the numeral says.

6

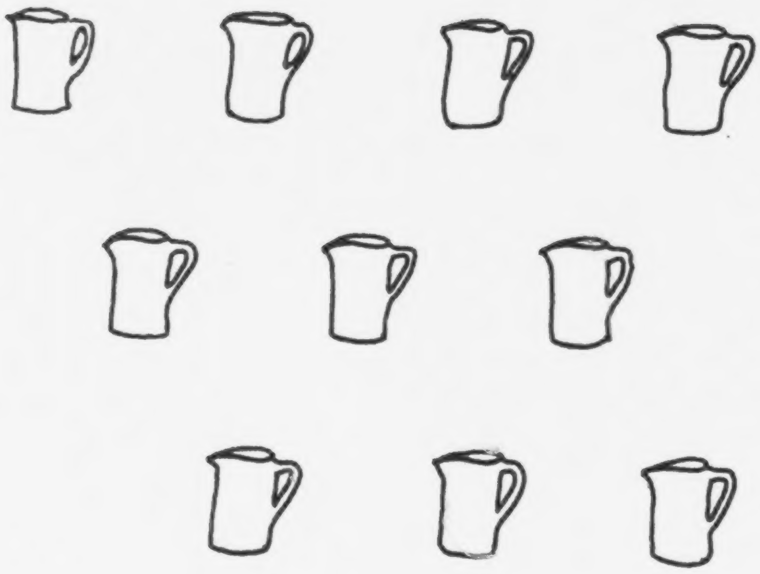


Would you accept correct responses to items like these as evidence that a child can select a set of objects the number of which is represented by a given numeral?

TEACHER READS:

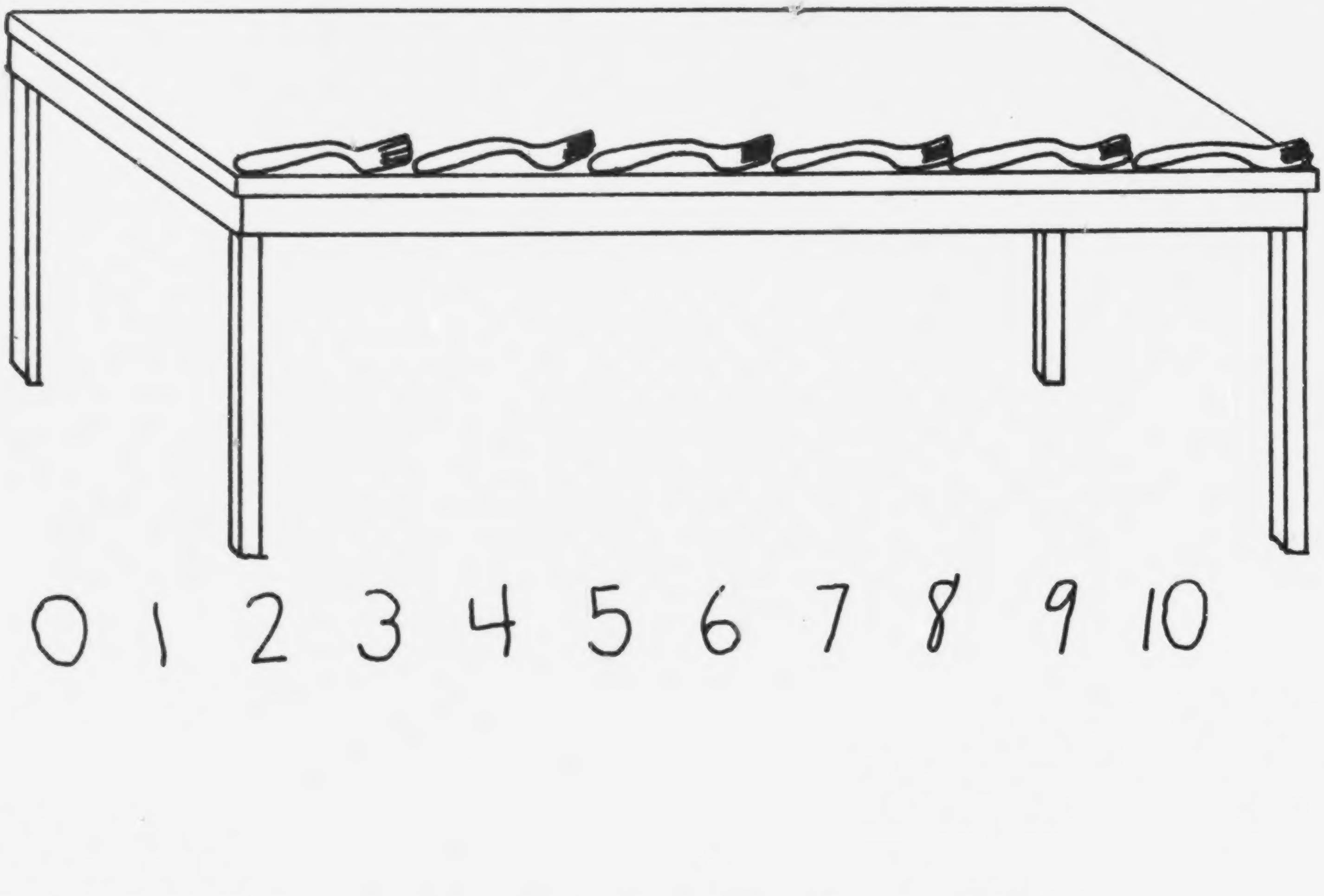
Look at the numeral on this page. Draw a circle to show as many pitchers as this numeral says.

4



TEACHER READS:

Draw a circle around the numeral that tells how many forks long the table is.

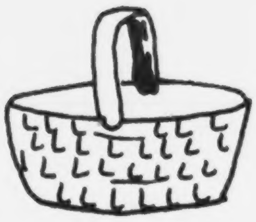


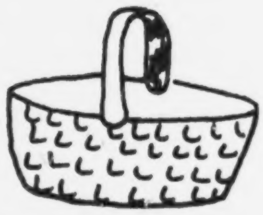
0 1 2 3 4 5 6 7 8 9 10

Would you accept correct responses to items like these as evidence that a child can select the appropriate numeral to represent a given set of objects?

TEACHER READS:

Draw a circle around the numeral that tells how many baskets there are on this page.

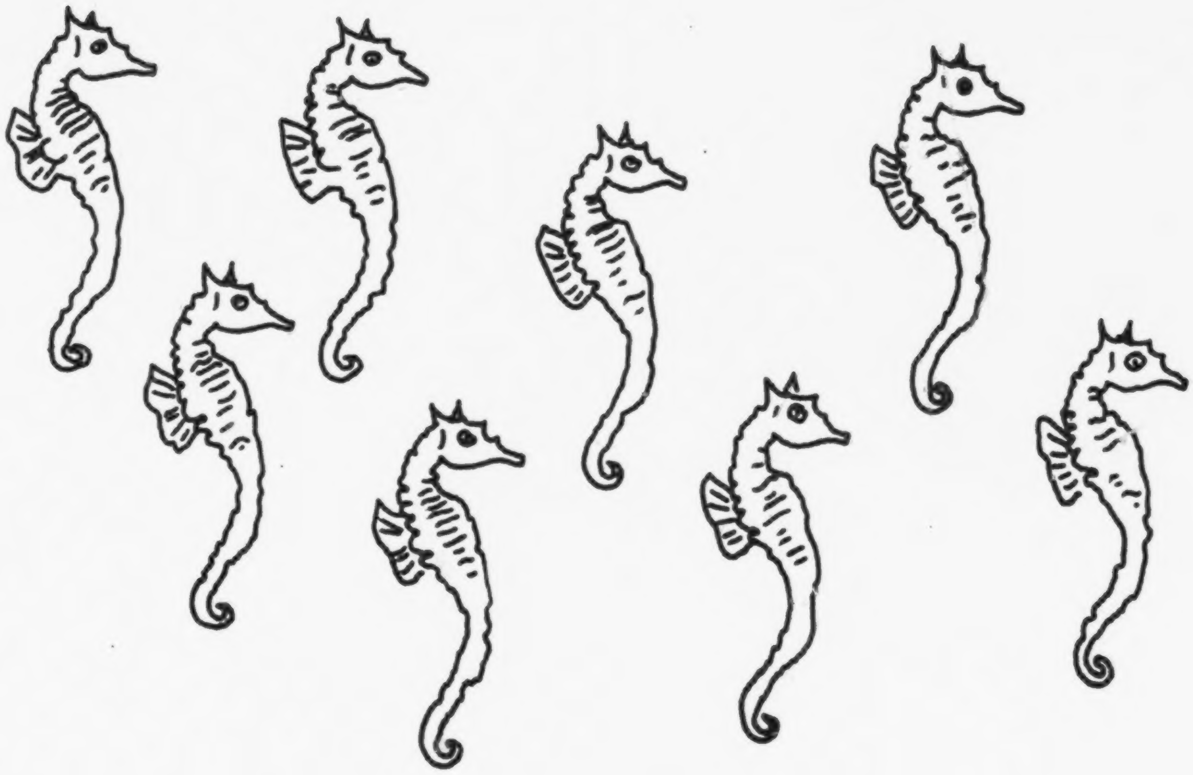




0 1 2 3 4 5 6 7 8 9 10

TEACHER READS:

Draw a circle around the numeral that tells how many seahorses there are on this page.



0 1 2 3 4 5 6 7 8 9 10

APPENDIX D

Item Statistics

133

Table 1

Item Statistics for Test I,
Classifying and Comparing Objects and Sets I

(Number responding = 45)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
		All items		
1	1.0000	.0000	.0000	.0000
2	1.0000	.0000	.0000	.0000
3	.8889	.3248	-3.7581	.3434
4	.3111	.6041	.8156	.7581
5	.8889	.4222	-2.8909	.4658
6	1.0000	.0000	.0000	.0000
7	.2000	-.0725	-11.6070	-.0727
8	1.0000	.0000	.0000	.0000
9	1.0000	.0000	.0000	.0000
10	.9778	.5462	-3.6797	.6521
11	.7333	.6346	-.9815	.8212
12	.6667	.8628	-.4992	1.7070
13	.6889	.9889	-.4983	6.6431
14	.9333	.4567	-3.2868	.5134
15	.9556	.8128	-2.0930	1.3954
16	.8444	.4610	-2.1971	.5195
17	.9556	.7145	-2.3811	1.0213
18	.9333	.8849	-1.6964	1.8994
19	.9778	.5462	-3.6797	.6521
20	.9333	.6708	-2.2378	.9044
21	.8000	.7317	-1.1502	1.0734
22	.9778	.8949	-2.2460	2.0047
23	.7111	.6409	-.8685	.8350
24	.7556	.3370	-2.0537	.3579

(continued)

Table 1 (continued)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
Each choice of Items 17-24				
17a	.9778	1.4464	.0000	.0000
17b	1.0000	.0000	.0000	.0000
17c	1.0000	.0000	.0000	.0000
17d	.9556	1.0299	.0000	.0000
18a	.9556	1.3308	.0000	.0000
18b	.9333	1.5579	.0000	.0000
18c	1.0000	.0000	.0000	.0000
18d	1.0000	.0000	.0000	.0000
19a	.9778	.9129	-2.2016	2.2367
19b	1.0000	.0000	.0000	.0000
19c	1.0000	.0000	.0000	.0000
19d	1.0000	.0000	.0000	.0000
19e	1.0000	.0000	.0000	.0000
20a	.9778	1.4464	.0000	.0000
20b	.9778	.9129	-2.2016	2.2367
20c	.9556	1.0299	.0000	.0000
20d	1.0000	.0000	.0000	.0000
20e	1.0000	.0000	.0000	.0000
20f	1.0000	.0000	.0000	.0000
21a	.9778	.7392	-2.7190	1.0976
21b	.9333	1.3268	.0000	.0000
21c	.8222	.3699	-2.4974	.3982
21d	.9333	1.4315	.0000	.0000
22a	.9778	2.0186	.0000	.0000
22b	.9778	2.0186	.0000	.0000
22c	.9778	2.0186	.0000	.0000
22d	.9778	2.0186	.0000	.0000
23a	.9333	.4888	-3.0709	.5603
23b	.9778	-.0284	70.6911	-.0284
23c	.9111	.2993	-4.5025	.3137
23d	.9333	1.1173	.0000	.0000
23e	.8889	.7469	-1.6342	1.1233

(continued)

Table 1 (continued)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
Each choice of Items 17-24				
24a	1.0000	.0000	.0000	.0000
24b	1.0000	.0000	.0000	.0000
24c	.9778	.7392	-2.7190	1.0976
24d	.8667	.3356	-3.3102	.3562
24e	.8444	.7498	-1.3509	1.1331
24f	1.0000	.0000	.0000	.0000

Table 2

Item Statistics for Test II,
Classifying and Comparing Objects and Sets II

(Number responding = 45)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
		All items		
1	.9556	.5629	3.0223	.6811
2	1.0000	.0000	.0000	.0000
3	.7111	.4568	-1.2184	.5136
4	.5111	-.0379	.7348	-.0379
5	.8222	.9584	-.9640	3.3567
6	.8667	.8072	-1.3761	1.3674
7	.5556	.2924	-.4779	.3057
8	.9111	.7516	-1.7929	1.1396
9	1.0000	.0000	.0000	.0000
10	1.0000	.0000	.0000	.0000
11	.8222	.7822	-1.1811	1.2556
12	.5778	.3937	-.4984	.4283
13	.7111	.7253	-.7675	1.0535
14	.7556	1.0349	.0000	.0000
15	.8000	.7808	-1.0778	1.2499
16	.8667	1.1267	.0000	.0000
17	.9556	.7258	-2.3440	1.0551
18	.9778	.1380	-14.5661	.1393
19	.7778	.5019	-1.5236	.5803
20	.7333	.2900	-2.1480	.3030
21	.9778	.2824	-7.1175	.2944
22	.8889	.4753	-2.5680	.5403
23	.9333	.0512	-29.3009	.0513
24	.8444	.6941	-1.4593	.9641

(continued)

137

Table 2 (continued)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
Each choice of Items 17-24				
17a	.9778	1.2280	.0000	.0000
17b	.9556	.7916	-2.1492	1.2955
17c	1.0000	.0000	.0000	.0000
17d	1.0000	.0000	.0000	.0000
18a	.9778	.7017	-2.8643	.9849
18b	1.0000	.0000	.0000	.0000
18c	1.0000	.0000	.0000	.0000
18d	1.0000	.0000	.0000	.0000
19a	.9333	.4309	-3.4839	.4775
19b	.9778	1.2280	.0000	.0000
19c	1.0000	.0000	.0000	.0000
19d	1.0000	.0000	.0000	.0000
19e	.8444	.8553	-1.1843	1.6506
20a	.9778	.1754	-11.4572	.1782
20b	.7778	.4989	-1.5328	.5756
20c	1.0000	.0000	.0000	.0000
20d	1.0000	.0000	.0000	.0000
20e	.9778	.7017	-2.8643	.9849
20f	.9778	1.2280	.0000	.0000
21a	1.0000	.0000	.0000	.0000
21b	1.0000	.0000	.0000	.0000
21c	1.0000	.0000	.0000	.0000
21d	.9778	.2469	-8.1418	.2547
22a	.9333	1.4894	.0000	.0000
22b	.9556	1.6406	.0000	.0000
22c	.9333	1.2917	.0000	.0000
22d	.9333	1.2917	.0000	.0000
23a	1.0000	.0000	.0000	.0000
23b	1.0000	.0000	.0000	.0000
23c	.9778	.2469	-8.1418	.2547
23d	1.0000	.0000	.0000	.0000
23e	.9556	.2785	-6.1092	.2899

(continued)

Table 2 (continued)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
Each choice of Items 17-24				
24a	1.0000	.0000	.0000	.0000
24b	.9333	.5009	-2.9970	.5787
24c	1.0000	.0000	.0000	.0000
24d	.9778	.2469	-8.1418	.2547
24e	.9333	.3032	-4.9515	.3181
24f	1.0000	.0000	.0000	.0000

Table 3

Item Statistics for Test III, Equalizing Objects on Length

(Number responding = 37)

Item No.	Response	Difficulty	Item-test Correlation	X_{50}	Beta
1	Equal.	.5135	.7947	-.0426	1.3093
	Wrong	.2432	-.6271	-1.1098	-.8050
	Comp.	.2432	-.3848	-1.8084	-.4169
2	Equal.	.0270	.5660	3.4036	.6865
	Wrong	.8649	.4094	-2.6927	.4488
	Comp.	.1081	-.6691	-1.8481	-.9004
3	Equal.	.5135	.6573	-.0515	.8722
	Wrong	.3243	-.4976	-.9157	-.5763
	Comp.	.1622	-.3387	-2.9096	-.3600
4	Equal.	.6216	.6284	-.4929	.8078
	Wrong	.2162	-.4021	-1.9522	-.4392
	Comp.	.1622	-.4933	-1.9981	-.5670
5	Equal.	.5676	.5139	-.3312	.5990
	Wrong	.1622	-.6478	-1.5215	-.8503
	Comp.	.2703	-.1301	-4.7024	-.1313
6	Equal.	.6216	.6727	-.4604	.9093
	Wrong	.1622	-.5791	-1.7019	-.7103
	Comp.	.2162	-.3878	-2.0246	-.4207
7	Equal.	.4865	.7379	.0459	1.0932
	Wrong	.2703	-.4231	-1.4463	-.4670
	Comp.	.2432	-.4925	-1.4131	-.5659
8	Equal.	.6216	.7503	-.4128	1.1350
	Wrong	.2703	-.6652	-.9200	-.8909
	Comp.	.1081	-.3514	-3.5189	-.3754
9	Equal.	.6757	.9546	-.4773	3.2051
	Wrong	.1351	-.7780	-1.4171	-1.2381
	Comp.	.1892	-.6439	-1.3681	-.8415

(continued)

Table 3 (continued)

Item No.	Response	Difficulty	Item-test Correlation	X_{50}	Beta
10	Equal.	.5946	.9710	-.2465	4.0613
	Wrong	.1892	-.8463	-1.0409	-1.5886
	Comp.	.2162	-.5028	-1.5614	-.5816
11	Equal.	.5946	.8406	-.2848	1.5516
	Wrong	.1892	-.9553	-.9221	-3.2303
	Comp.	.2162	-.2296	-3.4188	-.2359
12	Equal.	.7027	.9230	-.5766	2.3983
	Wrong	.1351	-.9331	-1.1815	-2.5952
	Comp.	.1622	-.4761	-2.0702	-.5414
13	Equal.	.5405	.7493	-.1359	1.1315
	Wrong	.1351	-.9331	-1.1815	-2.5952
	Comp.	.3243	-.2632	-1.7312	-.2728
14	Equal.	.5676	.7496	-.2270	1.1327
	Wrong	.1351	-.9331	-1.1815	-2.5952
	Comp.	.2973	-.2658	-2.0024	-.2757
15	Equal.	.6486	.8644	-.4415	1.7192
	Wrong	.1081	-.8961	-1.3801	-2.0185
	Comp.	.2432	-.4925	-1.4131	-.5659
16	Equal.	.5405	.7705	-.1321	1.2089
	Wrong	.1892	-.8463	-1.0409	-1.5886
	Comp.	.2703	-.2321	-2.6373	-.2386
17	Equal.	.5135	.7736	-.0438	1.2207
	Wrong	.1351	-.8167	-1.3498	-1.4155
	Comp.	.3514	-.3531	-1.0808	-.3774
18	Equal.	.7027	.8134	-.6542	1.3985
	Wrong	.0541	-.5086	-3.1590	-.5908
	Comp.	.2432	-.7213	-.9649	-1.0413
19	Equal.	.6757	.6616	-.6886	.8824
	Wrong	.1622	-.4933	-1.9981	-.5670
	Comp.	.1622	-.4761	-2.0702	-.5414
20	Equal.	.5135	.7736	-.0438	1.2207
	Wrong	.1622	-.8710	-1.1316	-1.7728
	Comp.	.3243	-.2632	-1.7312	-.2728

(continued)

Table 3 (continued)

Item No.	Response	Difficulty	Item-Test Correlation	X_{50}	Beta
21	Equal.	.7297	.8181	-.7481	1.4223
	Wrong	.0541	-.7391	-2.1740	-1.0971
	Comp.	.2162	-.6465	-1.2143	-.8475
22	Equal.	.6486	.9667	-.3948	3.7748
	Wrong	.1351	-.7780	-1.4171	-1.2381
	Comp.	.2162	-.6465	-1.2143	-.8475

Table 4

Item Statistics for Test IV,
Equalizing Sets on Numerousness

(Number responding = 38)

Item No.	Response	Difficulty	Item-test Correlation	X_{50}	Beta
1	Equal.	.5526	.7981	-.1658	1.3245
	Wrong	.2632	-.7744	-.8182	-1.2240
	Comp.	.1842	-.2361	-3.8091	-.2430
2	Equal.	.6316	.6732	-.4991	.9105
	Wrong	.1842	-.7174	-1.2538	-1.0297
	Comp.	.1842	-.2361	-3.8091	-.2430
3	Equal.	.7368	.7364	-.8604	1.0885
	Wrong	.2105	-.8380	-.9601	-1.5359
	Comp.	.0526	.0142	114.2933	.0142
4	Equal.	.8421	.6493	-1.5450	.8537
	Wrong	.1316	-.6312	-1.7728	-.8138
	Comp.	.0263	-.3601	-5.3818	-.3860
5	Equal.	.4474	.9367	.1413	2.6749
	Wrong	.2895	-.7549	-.7351	-1.1511
	Comp.	.2632	-.3439	-1.8426	-.3662
6	Equal.	.7632	.7879	-.9093	1.2797
	Wrong	.1842	-.9192	-.9785	-2.3338
	Comp.	.0526	.0142	114.2933	.0142
7	Equal.	.4474	.9053	.1461	2.1317
	Wrong	.3684	-.6732	-.4991	-.9105
	Comp.	.1842	-.3914	-2.2982	-.4253
8	Equal.	.3947	.8402	.3178	1.5492
	Wrong	.3947	-.5661	-.4716	-.6868
	Comp.	.2105	-.3655	-2.2013	-.3927
9	Equal.	.7105	.5616	-.9882	.6787
	Wrong	.2632	-.6225	-1.0180	-.7953
	Comp.	.0263	.1818	10.6580	.1849

Table 4 (continued)

Item No.	Response	Difficulty	Item-test Correlation	X_{50}	Beta
10	Equal.	.8421	.8891	-1.1282	1.9429
	Wrong	.1316	-.9218	-1.2139	-2.3775
	Comp.	.0263	-.2924	-6.6288	-.3057
11	Equal.	.4737	.9688	.0681	3.9087
	Wrong	.1316	-1.0187	.0000	.0000
	Comp.	.3947	-.4373	-.6105	-.4863
12	Equal.	.8158	1.0278	.0000	.0000
	Wrong	.1579	-.9748	-1.0291	-4.3706
	Comp.	.0263	-.6311	-3.0710	-.8135
13	Equal.	.3421	.7415	.5485	1.1051
	Wrong	.2368	-.5335	-1.3430	-.6308
	Comp.	.4211	-.2753	-.7236	-.2864
14	Equal.	.4211	.9716	.2050	4.1059
	Wrong	.2368	-.8549	-.8381	-1.6479
	Comp.	.3421	-.3163	-1.2861	-.3334
15	Equal.	.7632	.5335	-1.3430	.6308
	Wrong	.1842	-.4535	-1.9835	-.5088
	Comp.	.0526	-.4090	-3.9607	-.4482
16	Equal.	.4737	.8754	.0754	1.8106
	Wrong	.1842	-.8260	-1.0889	-1.4656
	Comp.	.3421	-.3500	-1.1620	-.3737
17	Equal.	.3947	.8402	.3178	1.5492
	Wrong	.2895	-.7187	-.7722	-1.0335
	Comp.	.3158	-.2184	-2.1959	-.2238
18	Equal.	.7105	.7670	-.7235	1.1953
	Wrong	.1579	-.7863	-1.2757	-1.2728
	Comp.	.1316	-.3406	-3.2855	-.3622
19	Equal.	.6316	.6732	-.4991	.9105
	Wrong	.2632	-.7744	-.8182	-1.2240
	Comp.	.1053	-.0060	-209.7269	-.0060
20	Equal.	.4737	.7404	.0892	1.1015
	Wrong	.1579	-.8035	-1.2485	-1.3496
	Comp.	.3684	-.2677	-1.2554	-.2778

(continued)

Table 4 (continued)

Item No.	Response	Difficulty	Item-test Correlation	X_{50}	Beta
21	Equal.	.6842	.7994	-.5998	1.3307
	Wrong	.2105	-.9239	-.8708	-2.4153
	Comp.	.1053	-.0967	-12.9461	-.0972
22	Equal.	.7632	.9352	-.7661	2.6419
	Wrong	.1842	-.9657	-.9313	-3.7218
	Comp.	.0526	-.2936	-5.5176	-.3071

Table 5

Item Statistics for Test V, Ordering More Than Two Objects and Sets

(Number responding = 38)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
		All items		
1	.9737	.1463	-13.2487	.1479
2	.9474	.8895	-1.8211	1.9468
3	.9474	.8895	-1.8211	1.9468
4	.9737	.2621	-7.3946	.2716
5	.8421	.5735	-1.7493	.7000
6	.3421	.1266	3.2139	.1276
7	.7368	.6414	-.9879	.8360
8	.3947	.7534	.3544	1.1457
9	.4737	.6968	.0947	.9715
10	.5789	.4373	-.4555	.4863
11	.6053	.6230	-.4286	.7964
12	.4737	.7323	.0901	1.0754
13	.6842	.4601	-1.0423	.5182
14	.9737	.1463	-13.2487	.1479
15	.8421	.3391	-2.9579	.3605
16	.8421	.5735	-1.7493	.7000
17	.7368	.5981	-1.0594	.7463
18	.1842	.7207	1.2480	1.0396
19	.4211	.5562	.3562	.6692
20	.6316	.4250	-.7906	.4696
21	.5263	.5633	-.1172	.6817
22	.6053	.4028	-.6629	.4400
23	.5263	.5278	-.1251	.6213

Table 6

Item Statistics for Test VI,
Numbers and Numerals I

(Number responding = 38)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
		All items		
1	.7632	.7106	-1.0083	1.0099
2	.8684	.6883	-1.6256	.9489
3	.6579	.4269	-.9528	.4721
4	.5263	.2766	-.2386	.2879
5	.7368	.5965	-1.0623	.7431
6	.7895	.7984	-1.0078	1.3259
7	.8947	1.2871	.0000	.0000
8	.9474	.6724	-2.4092	.9083
9	.5263	.3445	-.1916	.3669
10	.7105	.8524	-.6510	1.6303
11	.9474	1.5520	.0000	.0000
12	.8684	1.0891	.0000	.0000
13	.8158	.7891	-1.1399	1.2845
14	.8684	1.3844	.0000	.0000
15	.6053	.5014	-.5325	.5795
16	.9211	1.5158	.0000	.0000
17	.6842	.7126	-.6729	1.0157
18	.8684	1.3844	.0000	.0000
19	.8421	1.0849	.0000	.0000
20	.9211	1.5158	.0000	.0000
21	.6842	.5481	-.8749	.6553
22	.8684	.7305	-1.5317	1.0698
23	.8947	1.5341	.0000	.0000
24	.8158	.9750	-.9225	4.3881
25	.9211	1.3323	.0000	.0000
26	.7105	.8129	-.6826	1.3960
27	.8947	.6942	-1.8036	.9646
28	.9737	.9239	-2.0976	2.4143

Table 7

Item Statistics for Test VII,
Numbers and Numerals II

(Number responding = 37)

Item No.	Difficulty	Item-test Correlation	X_{50}	Beta
All items				
1	.7027	.7283	-.7307	1.0629
2	.8378	.7872	-1.2520	1.2765
3	.7297	1.0082	.0000	.0000
4	.6486	.1948	-1.9593	.1986
5	.7568	.2921	-2.3821	.3055
6	.8919	1.2432	.0000	.0000
7	.9189	.9017	-1.5502	2.0856
8	.9189	.8443	-1.6557	1.5754
9	.7297	.7345	-.8332	1.0824
10	.5676	.8681	-.1960	1.7489
11	.7838	.7661	-1.0248	1.1919
12	.3243	.6281	.7255	.8071
13	.7838	1.0161	.0000	.0000
14	.7838	1.1191	.0000	.0000
15	.8108	1.1303	.0000	.0000
16	.8108	1.0984	.0000	.0000
17	.5946	.8456	-.2831	1.5842
18	.7297	.7606	-.8046	1.1715
19	.8378	.2602	-3.7881	.2695
20	.3514	.7003	.5450	.9811
21	.8649	.9859	-1.1182	5.8875
22	.7297	.5651	-1.0831	.6849
23	.9730	1.1844	.0000	.0000
24	.8649	1.1843	.0000	.0000
25	.9189	1.0165	.0000	.0000
26	.7568	.5951	-1.1694	.7405
27	.9189	1.2175	.0000	.0000
28	.5946	.6232	-.3841	.7968